

# A Dynamic, Stochastic, Computational Model of Preference Reversal Phenomena

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Preference orderings among a set of options may depend on the elicitation method (e.g., choice or pricing); these preference reversals challenge traditional decision theories. Previous attempts to explain these reversals have relied on allowing utility of the options to change across elicitation methods by changing the decision weights, the attribute values, or the combination of this information—still, no theory has successfully accounted for all the phenomena. In this article, the authors present a new computational model that accounts for the empirical trends without changing decision weights, values, or combination rules. Rather, the current model specifies a dynamic evaluation and response process that correctly predicts preference orderings across 6 elicitation methods, retains stable evaluations across methods, and makes novel predictions regarding response distributions and response times.

*Keywords:* preference reversals, anchoring and adjustment, pricing, stochastic choice models, dynamic decision models

Intuitively, the concept of preference seems very clear and natural, but under close scientific scrutiny, this concept becomes quite complex and multifaceted. Theoretically, *preference* is an abstract relation between two options: When an individual is presented with Options A and B, it is assumed that he or she either prefers A to B or prefers B to A (or is indifferent between A and B). It is important to recognize, however, that this abstract relation is a psychological construct that must be operationalized or measured by some observable behavior (Garner, Hake, & Eriksen, 1956).

Several standard methods have been used by decision theorists for the measurement of preference (cf. Keeney & Raiffa, 1976; Luce, 2000; Raiffa, 1968). Perhaps the most common way is a choice procedure in which an individual is asked to choose among options and choice frequency is used to rank order preferences. A more convenient method is to obtain a single value for each option by asking an individual to state a price or dollar value that is considered equivalent to an option, called a *certainty equivalent* (CE); in this case, the price is used to rank order preferences. Variations on the pricing method include asking for a buying price

or how much money one is willing to pay (WTP) to acquire an option or asking how much money one is willing to accept (WTA) to forego or sell an option. Finally, researchers often measure preference by stating the probability of winning a bet that is considered equivalent to another option, called a *probability equivalent* (PE); in this case, the probability of winning is used to rank order preferences.

According to classic utility theories, the standard methods for measuring preference (i.e., choice, CE, and PE) should agree and produce the same rank order of preference over options (see Keeney & Raiffa, 1976; Luce, 2000; Raiffa, 1968).<sup>1</sup> This conclusion follows from two key assumptions (see Figure 1): (a) A single *utility mapping* transforms options into utilities, and (b) a *monotonic response mapping* transforms these utilities into observed measurements. Thus, Option A is chosen more frequently than Option B only if the utility is greater for Option A compared with B, and the latter is true only if the CE and PE are greater for Option A compared with B.

During the past 30 years of empirical research on the measurement of preference, researchers have found systematic preference reversals among the standard measurement methods. In other words, the rank order produced by one method does not agree with the rank order produced by a second method (for reviews, see Seidl, 2001; Slovic & Lichtenstein, 1983; and Tversky, Slovic, & Kahneman, 1990). The “original” preference reversals, between choices and prices, were reported by Lichtenstein and Slovic (1971) and Lindman (1971), who used gambles as stimuli; the phenomenon has subsequently been repeated with a variety of controls and conditions (e.g., Grether & Plott, 1979). The occurrence of these reversals seems to depend specifically on a set in which a low-variance gamble (L) offers a high probability of

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A substantial portion of this work was included in Joseph G. Johnson’s doctoral dissertation and presented at his 2004 Einhorn New Investigator Award acceptance presentation. This work was supported by National Science Foundation Methodology, Measurement, and Statistics Grant SES-0083511, received while Joseph G. Johnson was at Indiana University Bloomington, and by National Institute of Mental Health National Research Service Award No. MH14257 to the University of Illinois. We thank David Budescu, Sarah Lichtenstein, Tony Marley, Mike Regenwetter, Jim Sherman, Richard Shiffrin, Paul Slovic, Jim Townsend, and Wayne Winston for helpful comments on this research.

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<sup>1</sup> According to standard economic theory, discrepancies between buying and selling prices may occur because of differences in wealth; however, this effect is too small to account for the reversals reviewed herein (see Harless, 1989).

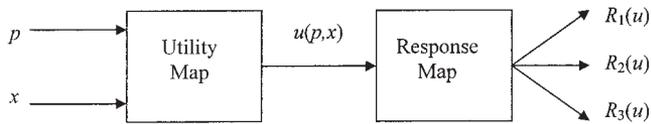


Figure 1. Information integration diagram. Properties of an option (e.g., probabilities and payoffs) are mapped into a utility, and then this utility is mapped into an overt response (cf. Anderson, 1996).  $p$  = probability;  $x$  = value;  $u$  = utility function;  $R$  = response function.

winning a modest amount and a high-variance gamble (H) offers a moderate probability of winning a large amount.

Shortly after this first preference reversal research, Birnbaum and Stegner (1979) found preference reversals between selling prices and buying prices (see also Birnbaum & Sutton, 1992; Birnbaum & Zimmermann, 1998), a finding that is closely related to discrepancies found in economics between WTP and WTA (see Horowitz & McConnell, 2002, for a review). Finally, inconsistencies were found between preferences inferred from PEs and CEs (Hershey & Schoemaker, 1985; Slovic, Griffin, & Tversky, 1990).

These results call into question at least one of the two basic assumptions described above: Either (a) different utility mappings are used to map options into utilities or (b) the response mappings are not all monotonically related to utilities. To account for these findings, most theorists adopt the first hypothesis—that is, most explanations continue to assume a monotonic relation between the utilities and the measurements, but different utilities are permitted to be used for each measurement method. This reflects the intuitive idea that utilities are context dependent and constructed to serve the purpose of the immediate task demands (Payne, Bettman, & Johnson, 1992; Slovic, 1995). According to this *constructed utility* hypothesis, if preferences reverse across measurement methods, then this implies that the underlying utilities must have changed in a corresponding manner.

The purpose of this article is to present an alternative theory that retains the first assumption of a single mapping from options to utilities but rejects the second assumption that the measurements are all monotonically mapped into responses. The proposed theory provides a dynamic, stochastic, and computational model of the response process underlying each measurement method, which may be nonmonotonic with utility. We argue that this idea provides a straightforward explanation for all of the different types of preference reversals in a relatively simple manner while retaining a consistent underlying utility structure across all preference measures.

### Theories Assuming Context-Dependent Utility Mappings

If one assumes that the locus of context effects is in mapping from options to utilities, each method of measuring preferences requires a separate utility theory. In general, utility mappings are formalized in terms of three factors: (a) the values assigned to the outcomes of an option; (b) the weights assigned to the outcomes, which depend on the probabilities of the outcomes or the importance of the attributes; and (c) the combination rule used to combine the weights and values. Thus, previous explanations have required (at least) one of three general modifications of utilities across contexts: alterations of (a) the value of each outcome and/or (b) the weight given to each outcome and/or (c) the integration of this information.

Researchers attempting to account for preference reversals between elicitation methods have relied primarily on context-dependent changes in these components of the utility function. Initially, Birnbaum and Stegner (1979) and other colleagues (e.g., Birnbaum, Coffey, Mellers, & Weiss, 1992) allowed the rank-dependent (configural) weights of payoffs to change across buying, selling, and neutral points of view. Later, Tversky, Sattath, and Slovic (1988) proposed changes in contingent weighting of probability versus payoff attributes across tasks. Kahneman (e.g., Kahneman, Knetsch, & Thaler, 1990) suggested changes in valuation for WTA and WTP. Mellers and colleagues (e.g., Mellers, Chang, Birnbaum, & Ordoñez, 1992) have argued for changes in the combination rule between price and choice. Finally, Loomes and Sugden (1983) pointed out that preference reversals may reflect intransitive preferences caused by a regret utility function. Nevertheless, we believe that tuning the utility function has not provided a completely adequate account of all the empirically supported preference reversals. None of these research programs has been shown to account for all the various types of preference reversals among choice, CE, pricing, and PE methods.

### Theories Assuming Context-Dependent Response Mappings

The inadequacy of past theories may lie precisely in their sole reliance on changes in the utility representation. One might realize a simpler explanation by retaining invariant utility mappings and focusing on the response mapping. After all, it seems natural that changes in the response method affect the response mapping (rather than the utility mapping). It is likely that both mappings are context dependent; however, we plan to examine the extent to which all of the results can be explained solely in terms of a nonmonotonic response process. In our case, the utility computation for an option is stable and consistent across tasks, whereas the response process is responsible for differences in elicited preference orders.

Our focus on the response process, or mapping of internal utilities to an overt response, has predecessors that date to the original study credited for revealing preference reversals. Initially, Lichtenstein and Slovic (1971) proposed an anchoring and adjustment theory, which has also been the basis of other processing accounts of preference reversals (Goldstein & Einhorn, 1987; Schkade & Johnson, 1989). Most generally, these theories assume that choices reflect the “true” utility structure but that, when giving a value response (e.g., price), individuals attempt to recover these utility values—susceptible to a systematic bias. In particular, the theories assume that when a decision maker states a price for a gamble, he or she “anchors” on the highest possible outcome. Then, to determine the reported price, the decision maker adjusts (downward) from this anchor toward the true, underlying utility value. If this adjustment is insufficient, then preference reversals can occur if the gambles have widely disparate outcome ranges—as do the prototypical L and H gambles, which produce reversals between choice and pricing.

The current model shares conceptual underpinnings with these earlier process models, but it is also distinctly different. First, it offers a more comprehensive account of preference reversals across all aforementioned elicitation methods. Second, the proposed theory formalizes an exact dynamic mechanism for such an adjustment process, independent of the empirical data to be pre-

dicted. Third, specific predictions differentiate the theories, which we identify later. Finally, ours is the only applicable theory that has been formulated specifically for deriving *response distributions* (as opposed to simply central tendencies) as well as predictions regarding deliberation time.

### Computational Modeling: Application to Preference Elicitation Methods

The present theory is a departure from the theoretical norm in that it does not view preference as a static relation but instead conceives of preference as a dynamic and stochastic process that evolves across time. The present work generalizes and extends earlier efforts to develop a dynamic theory of preference called *decision field theory* (DFT; Busemeyer & Goldstein, 1992; Busemeyer & Townsend, 1993; Townsend & Busemeyer, 1995). Although the present work builds on these earlier ideas, it substantially generalizes these principles, providing a much broader range of applications. In this section, we first introduce the experimental paradigm and present a generalization of the previous DFT model for binary choice among gambles. Second, we extend the choice model to accommodate the possibility of an indifference response, which is a crucial step for linking choice to other measures of preference. Third, we present a matching model for valuation measures (prices and equivalence values) of preference. The matching model is driven by the choice model, thus producing a comprehensive, hierarchical process model of various preferential responses.

Before we begin, it is imperative that we mention the distinction between the conceptual process of our model and the mathematical predictions derived from this process. We conceptualize the choice deliberation process as sequential sampling of information about the choice options, described below (see also Busemeyer & Townsend, 1993). This sequential sampling process has received considerable mathematical treatment (e.g., Bhattacharya & Waymire, 1990; Diederich & Busemeyer, 2003), which has provided theorems deriving mathematical formulas for precisely computing choice probabilities and deliberation times. Thus, whereas our theory postulates a sequential sampling process, we use the mathematically derived formulas to compute predictions for the results of this process. Because of limitations of space, we restrict ourselves primarily to an intuitive description of the process. Complete derivations and proofs of the mathematical formulas can be found in Appendix A and the references provided throughout this section.

#### DFT Model of Binary Choice

When one makes a choice, one rarely makes a decision instantly when the available options are presented. Rather, one may fluctuate in momentary preference between the options until finally making a choice. We conceptualize choice as exactly this—fluctuation across a preference continuum where the endpoints represent choice of either option. During a single choice trial, we assume that a series of evaluations are generated for each option, as if the decision maker were imagining the outcomes of many simulated plays from each gamble. The integration of these evaluations drives the accumulation of preference back and forth between the options over time. At some point, this deliberation process must stop and produce a response; DFT assumes that there

exists a threshold, a level at which an option is determined good enough to make a choice.

First, how are the momentary evaluations generated? Let us apply the DFT choice process to two classic example gambles, here labeled F and G, described as follows: Suppose numbers are randomly drawn from a basket. If any number up through 35 (inclusive of 36 numbers) is drawn, the outcome \$4 would result for Gamble F, and no gain would result otherwise. For Gamble G, if any number 1 through 11 (inclusive) is drawn, the gamble would pay \$16, but it would pay nothing on numbers 12 and higher. Each moment in the DFT choice process is akin to mentally sampling one of these numbers, producing an affective reaction to the imagined result. For example, perhaps a decision maker imagines drawing the number 20, which results in an evaluation of \$4 for Gamble F and \$0 for Gamble G. At the next moment, perhaps a different number is considered, and preferences are updated accordingly. Thus, the outcome probabilities dictate where attention shifts, but only the outcome values are used in determining the momentary evaluation. It is this sequential sampling of evaluations driven by probabilities and outcomes, rather than the direct computation of expected values or utilities, that underlies the DFT choice process.

Each imagined event (e.g., number drawn) produces a comparison between the evaluations sampled from each option (e.g., gamble) at each moment in time. We symbolize the momentary evaluation for a Gamble F at time  $t$  as  $V_F(t)$  and the momentary evaluation for Gamble G at time  $t$  as  $V_G(t)$ . The momentary comparison of these two evaluations produces what is called the valence at time  $t$ :  $V(t) = V_F(t) - V_G(t)$ .

We can mathematically derive the theoretical mean of the sampling process from the expectation of the sample valence difference:

$$\mu = E[V(t)]. \tag{1}$$

This mean valence difference,  $\mu$ , will thus be positive if the evaluation  $V_F(t)$  is better than  $V_G(t)$ , on average over time, and negative when the average evaluations tend to favor Gamble G.

The uncertainty associated with Options F and G generates fluctuations in the decision maker's evaluations over time, so that at one point they may strongly favor one option, and at another point they may weakly favor the other option. Thus, we recognize that there is a great deal of fluctuation in  $V(t)$  over time. It is therefore theoretically important to mathematically derive the variance of the valence difference from the expectation:

$$\sigma_v^2 = E[V(t) - \mu]^2. \tag{2}$$

Finally, we use Equations 1 and 2 together to derive the crucial theoretical parameter for the DFT choice model, the discriminability index:

$$d = \frac{\mu}{\sigma_v}. \tag{3}$$

Intuitively, this ratio represents the expected tendency at any moment during deliberation to favor one option over the other, relative to the amount of overall variation in the evaluations about the options. As in signal detection theory (Green & Swets, 1966), the discriminability index is a theoretical measure—representing a summary of implicit samples from the distribution of evaluations—rather than a quantity directly experienced by the decision

maker. The ratio of mean and standard deviation has also appeared in other recent models of choice (see Erev & Baron, 2003; Weber, Shafir, & Blais, 2004).

Figure 2a illustrates the basic formal ideas of the sequential sampling process as a discrete Markov chain operating between two gambles, F and G. The circles in the figure represent different states of preference, ranging from a lower threshold (sufficient preference for choosing G), to zero (representing a neutral level of preference), to a symmetric upper threshold (sufficient preference for choosing F).<sup>2</sup> Each momentary evaluation either adjusts the preference state up a step (+Δ) toward the threshold for choosing F or down a step (−Δ) toward the threshold for choosing G. The step size, Δ, is chosen to be sufficiently small to produce a fine-grain scale that closely approximates a preference continuum. The threshold can also be defined via the number of steps times the step size (10Δ in Figure 2a).

The deliberation process begins in the neutral state of zero, unbiased toward either option. The transition probabilities for taking a positive or negative step at any moment, *p* or *q* in Figure 2a, respectively, can be derived via the theoretical parameters above and Markov chain approximations of the sequential sampling process (see Busemeyer & Townsend, 1992; Diederich & Busemeyer, 2003; and Appendix A for derivations):

$$p = \text{Pr}[\text{positive step}] = \frac{1}{2} + \frac{1}{2}\Delta \cdot d$$

$$q = \text{Pr}[\text{negative step}] = \frac{1}{2} - \frac{1}{2}\Delta \cdot d. \tag{4}$$

The final probability of choosing Gamble F is determined by the probability that the process will reach the right (positive) threshold

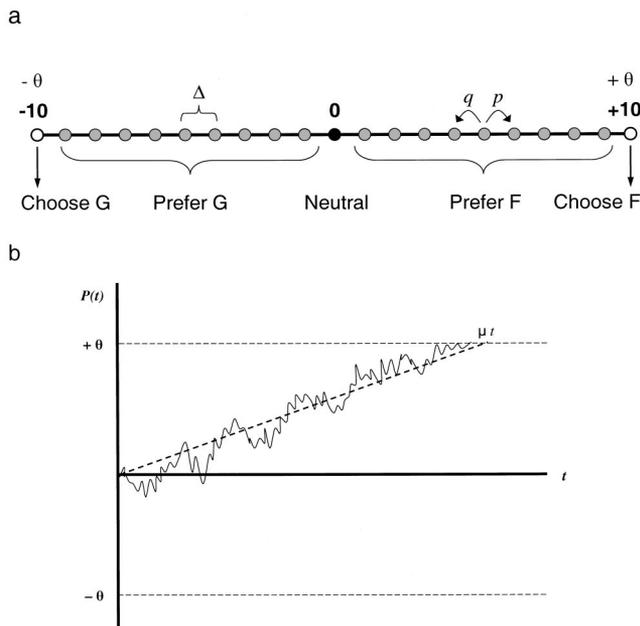


Figure 2. Representation of decision field theory choice model (a) as a discrete Markov chain and (b) as a Wiener diffusion process. Preference evolves over time toward a threshold,  $\theta$ , which we approximate with discrete states in Panel a, using probabilities  $p$  and  $q$  of moving a step size,  $\Delta$ , to each adjacent state. This process may produce a trajectory such as the jagged line in Panel b, producing a drift rate of  $\mu$  toward either threshold. F and G = gambles.  $t$  = time.

first; likewise, the final probability of choosing Gamble G is determined by the probability that the process will first reach the other threshold. Markov chain theorems also provide these choice probabilities (see Appendix A for details). A key advantage of DFT is that it also generates predictions regarding deliberation times (which also can be found via the equations in Appendix A; see Busemeyer & Townsend, 1993, and Diederich, 2003, for empirical applications).

The sequential sampling decision process can be viewed as a random walk with systematic drift, producing a trajectory such as the example shown in Figure 2b.<sup>3</sup> This figure plots the position of preference in Figure 2a—the momentary preference state between the thresholds—over time for a hypothetical choice trial. The transition probabilities in Equation 4 correspond to the probabilities of each increment or decrement of the solid line in Figure 2b. These drive the state toward a threshold boundary, at a mean rate  $\mu$  shown by the dashed line in Figure 2b. In the illustrated example, the sampling process results in a stochastic accumulation toward Gamble F, ultimately reaching the positive threshold and producing a choice of this gamble.

*Indifference Response*

Sometimes, when facing a choice between two options, one feels indifferent. That is to say that one would equally prefer to have the first option as the second—the options’ preferences are equal. This is precisely what CE and PE tasks ask for—a value that causes indifference when compared with a gamble. Such an indifference response can result from the DFT choice model through the inclusion of an assumption about when such a response may occur during deliberation.

What point in the DFT choice process corresponds to indifference, where neither option is (momentarily) preferred? We suppose that this point of indifference occurs whenever the momentary preference state is at zero or the neutral state. Recall that the choice process is assumed to start at zero, but after the process starts, we assume that whenever the fluctuating preference returns to the zero state (i.e., crosses the abscissa in Figure 2b), a decision maker may stop and respond as being indifferent between the two options. Specifically, we define the probability  $r$ , called the exit rate, as the probability that the process will stop with an indifference response whenever the preference state enters this neutral state (after the first step away from initial neutrality).

Altogether, the indifference choice model allows for three responses to occur. If the preference state reaches either threshold, then the corresponding option is selected. However, whenever the momentary preference enters the neutral state, then there is a probability of exiting and reporting indifference. Appendix A contains the formulas for computing the final probabilities for each of these three responses.

*Sequential Value-Matching (SVM) Process*

The DFT choice mechanism can provide choice probabilities among options, but the other response modes involve evaluating

<sup>2</sup> Note that the use of negative values for Gamble G and positive values for Gamble F, here and throughout, is arbitrary.

<sup>3</sup> See Laming (1968), Link and Heath (1975), Ratcliff (1978), Smith (1995), and Usher and McClelland (2001) for other applications of random walk models to decision making.

single options by reporting a value, such as a price, denoted  $C^*$ . Ultimately, we want to determine the probability that a particular value of  $C^*$  will be selected from some set of candidate values. We propose that this involves conducting an implicit search for a response value, successively comparing the target gamble with different values, until a value invokes a response of indifference. Thus, our SVM model involves two distinct modules: a candidate search module, and a comparison module. Essentially, the candidate search module recruits the comparison module (DFT indifference model) to specify the probability of selecting a particular value from the set of candidate values.

First, an intuitive explanation will help to illustrate the operation of each computational layer in the model. Consider the following specific gambles from Slovic et al. (1990): Gamble L offers \$4 with probability 35/36 and \$0 otherwise, denoted (35/36, \$4, \$0); Gamble H offers (11/36, \$16, \$0). Suppose the decision maker is asked to report a simple CE for Gamble H: “What amount to receive with certainty makes you indifferent between that amount and the gamble shown?” Imagine that  $C = \$8$  is first considered as a candidate, to elicit indifference between receiving  $C$  and Gamble H. Our model postulates that one of three mutually exclusive events occurs, depending on the decision maker’s true indifference point. If the value \$8 is a good or close estimate, then this value is very likely reported: CE = \$8. Second, if this value of  $C$  is too high, such that it is highly preferred to Gamble H, then the value must be decreased to elicit indifference—if one prefers \$8 to Gamble H, a lower amount, such as \$7, might be considered next. Third, if the gamble is preferred (the value of  $C$  is far too low), then the value must be increased. For illustration, assume this latter condition were the case. We hypothesize the decision maker would increment the value  $C$  and compare the new value of  $C$ , perhaps \$9, with Gamble H. This comparison could again result in an increase (to \$10) or a decrease (back to \$8) or end deliberation by reporting indifference as CE = \$9.

Figure 3a illustrates this SVM process for Gamble H, and Figure 3b shows how both layers of the process work together to find an indifference response. Consider first Figure 3a, which shows the potential transitions among various candidates for CE(H). We assume some finite set of candidates within a particular range, shown here as defined by the minimum and maximum outcomes of Gamble H. Leftward transitions in Figure 3a represent decreases in the candidate value, rightward transitions show increases, and downward transitions indicate selection of the candidate. Figure 3b conveys the relation among these transitions among candidates and the assessment of each particular value. If a candidate is selected in the top row, it is compared with the gamble using DFT, as shown in the middle panels. The result of this comparison determines whether the value is reported or whether another candidate is considered (bottom of Figure 3b). For example, if  $C_7$  is selected as a candidate, then the random walk will likely favor this value over the gamble. Thus, the value is decreased to  $C_6$ , and then this value is compared with the gamble. Eventually, the search will likely settle on  $C_4$ , where the random walk hovers around the point of indifference until the response is made (reporting CE =  $C_4$ ).

Technically, we define the underlying *comparison layer* (middle panels in Figure 3b) as a DFT indifference model, as described in the previous subsection, that is used to compare the candidate  $C$  and an arbitrary Gamble  $F$ . This process is parameterized in the indifference model by the assumption that, instead of comparing

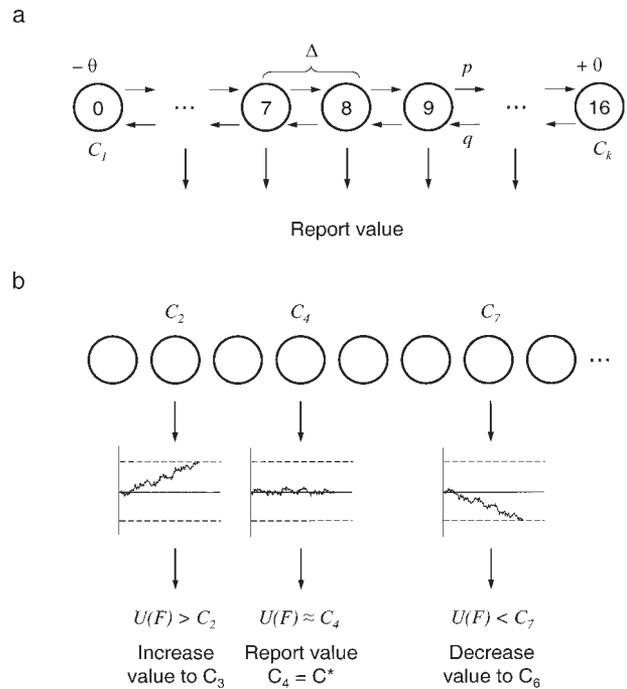


Figure 3. Representation of sequential value matching, showing (a) value search layer as a discrete Markov chain and (b) both search and comparison layers operating together. The search layer in Panel a is shown via discrete states as in Figure 2a, although now a response may occur from any state. In Panel b this search layer response is determined by the comparison layer (middle panels). The comparison layer operates as in Figure 2, using the input value selected by the search layer.  $\theta$  = threshold;  $\Delta$  = step size;  $p$  and  $q$  = probabilities;  $C$  = candidate value;  $k$  = number of candidate values;  $U$  = utility function;  $F$  = evaluated gamble.

Gamble  $F$  with Gamble  $G$ , one is comparing  $F$  with a sure thing value, the candidate  $C$ . Thus, we obtain from substitution in Equations 1 and 2

$$\mu = E[V_F(t) - V_C] \tag{1b}$$

and

$$\sigma_V^2 = E[V_F(t) - \mu]^2. \tag{2b}$$

One can see that now the comparison layer operates just like the DFT indifference model, where the transition probabilities are determined as before, driven by the discrimination index (Equation 4). When we include the exit rate, the probabilities of reaching either threshold or exiting with indifference are also computed as before. However, reaching either threshold does not determine selection of an option from a pair for a response, as it did before (cf. Figure 2). Only a comparison resulting in indifference corresponds to an overt response. In contrast, the positive threshold indicates strong preference for Gamble  $F$ , and the negative threshold signals strong preference for the candidate  $C$ , neither of which provides the desired indifference (equivalence) response. Either of these events entails using the search layer to adjust the candidate  $C$  to some new value, in search of  $C^*$  for which indifference does occur. In other words, the output probabilities of the comparison layer define the transition probabilities among values in the search layer.

The second process, the response layer or *search layer* (top panels in Figure 3b), is applied to adjust a candidate value up and down in search of indifference. First, we must declare a set of candidates for  $C^*$ . For simplicity, we assume the range of candidates for a CE is determined by the minimum and maximum outcomes of the evaluated gamble (as in Figure 3a for Gamble H). With this range established, we must next include how densely the set of candidates covers this range. As before, we assume that a finite set of  $k$  candidates is distributed across this range, such that the difference between any two candidates can again be written as a constant,  $\Delta = (\max - \min)/k$ . For illustration,  $k = 21$  in Figure 2a, and in Figure 3a,  $k = 17$  results in a step size ( $\Delta$ ) of \$1, which provides candidates with whole dollar amounts for Gamble H.

For value matching, we must also specify the initial state for the value search. Recall that for the choice as well as the comparison model, we assumed an unbiased initial state by beginning at zero. However, the zero or neutral state of the value search process is unknown—in fact, it is precisely what one is searching for. For now, we symbolize the initial candidate as the starting value  $C_0$ . The initial value is first input into the comparison layer, and then the comparison of this value  $C_0$  to Gamble F determines the initial transition probabilities for the value search layer. That is, the comparison layer defines the likelihood of either increases in the candidate, decreases in the candidate, or reporting of the current (first) candidate,  $C_0 = C^*$ . If either of the first two events occurs, then the appropriate neighboring value is compared next, and the process continues until an indifference response is made.

The primary dependent variable of the SVM model is the selection frequency for each of the candidate values. Specifically, we compute the distribution of response probabilities, denoted  $R_i$ , for each candidate  $C_i$ , indicating the probability that the associated comparison will result in the indifference response (see Appendix A for details).

The response probabilities for the matching values depend in part on where the search process starts—that is, the initial state,  $C_0$ . Again, this is distinct from the initial state of the comparison layer and choice model, which always start at neutral (in the current applications). Unlike the comparison layer of the model, the initial state of the value search,  $C_0$ , is not necessarily zero (i.e., \$0)—instead, assumptions must be stated about where the matching process begins within the range of candidates. We now show how the SVM model predicts CEs, buying prices, selling prices, and PEs, simply by specifying the initial candidate considered,  $C_0$  (see Figure 4).

**CEs.** In stating the CE, one is simply asked to report the price that elicits indifference between the price and a gamble. We assume that people are not immediately aware of their true indifference point, which must be discovered by the matching process. As mentioned, we assume that the candidate values for a CE are drawn from the range defined by the minimum and maximum gamble outcomes. One should not price a gamble higher than it could potentially be worth or lower than its minimum potential value. Given no prior information about where one's true indifference point lies, an unbiased estimate would be the middle of this range, or the average of the minimum and maximum candidate values. Therefore, as shown in the middle of Figure 4, we assume that the CE search process starts near the middle of candidate values to minimize the search required to find the true indifference point.

**Buying prices (WTP).** To specify the SVM model for predicting distributions of buying prices for a gamble, we again assume the range

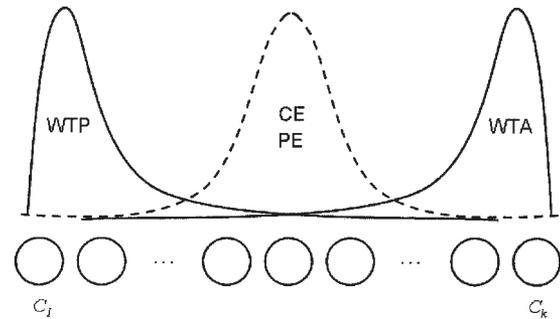


Figure 4. Stylized distributions of initial candidate values in the sequential value matching model. WTP = willingness to pay; CE = certainty equivalent; PE = probability equivalent; WTA = willingness to accept;  $C$  = candidate value;  $k$  = number of candidate values.

of candidates is determined by the gamble's outcomes. However, as shown in the left of Figure 4, we assume that the initial value,  $C_0$ , is skewed in favor of low prices, for reasons of competitive bidding. That is, one would attempt to pay as little as possible, increasing the price only as necessary. Although the initial state,  $C_0$ , is skewed, this occurs independently of the comparison process used to evaluate the gamble and each candidate price, which always assumes a neutral start in the current applications. In fact, there are no changes in the assumptions made about the comparison process. Thus, there will still be a tendency toward the value that causes true indifference, but the response probability distribution will exhibit the skew caused by the initial distribution in  $C_0$ .

**Selling prices (WTA).** The model for selling price responses is quite similar to that for buying prices. However, as shown in the right of Figure 4, we assume that the initial value,  $C_0$ , is skewed in favor of high prices, to maximize revenue.<sup>4</sup> When selling a gamble, one would attempt to charge as much as possible, decreasing the price only as necessary. Again, the comparison layer will drive the response value toward the true indifference point, but the response distribution will exhibit the inflation contained in the initial distribution.

**PEs.** In PE tasks, one is asked to state the probability of winning one gamble that makes it equally attractive to another gamble. That is, if presented with a target gamble,  $G$ , one is asked to provide the probability of winning ( $p$ ) in a reference Gamble F that produces indifference between the two gambles. To model this task, we simply assume a range in the SVM model defined by feasible probabilities, from zero to one, populated by equally spaced candidates  $p_i$ —using the same number of values,  $k$ . Then we use these candidate probabilities to determine the comparison values by filling in the missing outcome probability and computing the response distributions as before. For this matching procedure, we simply assume an unbiased initial candidate (i.e., starting near the middle of the relevant range of values,  $p_0 \approx .5$ ).

<sup>4</sup> Experimental manipulations could also help in determining initial values for any pricing measure (e.g., Schkade & Johnson, 1989). In fact, task instructions to state a maximum WTP and minimum WTA suggest our theoretical starting positions as well.

*Consistent Utility Mappings*

Hereafter, we apply the SVM model as described above, with additional assumptions and specific parameters. Consider a three-outcome gamble that offers  $x$  with probability  $p_x$ ,  $y$  with probability  $p_y$ , and  $z$  with probability  $p_z$ . First, it is assumed that the payoffs and probabilities of a gamble may be transformed into subjective evaluations for an individual. For each outcome  $i$ , we allow the outcome to be represented by its affective evaluation,  $u_i$ , and we allow the outcome probability to be transformed into a decision weight,  $w_i$ . However, according to DFT, the decision weight,  $w_i$ , represents the probability of evaluating outcome  $i$  of a gamble at any moment rather than an explicit weight used in computations.

Mathematically, this sampling assumption implies the following theoretical result for the mean valence difference appearing in Equation 1:

$$\mu = E[V(t)] = E[V_F(t)] - E[V_G(t)], \tag{5}$$

and

$$E[V_j(t)] = w_{jx} \times u_{jx} + w_{jy} \times u_{jy} + w_{jz} \times u_{jz}, \tag{6}$$

for  $j = \text{Gamble F or Gamble G}$ . If we assume statistically independent gambles, then this sampling assumption also implies the following theoretical result for the variance of the valence difference used in Equation 2:

$$\sigma_V^2 = E[V(t) - \mu]^2 = \sigma_F^2 + \sigma_G^2, \tag{7}$$

and

$$\sigma_j^2 = w_{jx} \times u_{jx}^2 + w_{jy} \times u_{jy}^2 + w_{jz} \times u_{jz}^2 - E[V_j(t)]^2, \tag{8}$$

for  $j = \text{Gamble F or Gamble G}$ . Finally, we define the utility of cash (e.g., price evaluations) similarly to be used in Equations 1b and 2b:

$$E[V_C(t)] = u_C. \tag{9}$$

Our assumption of consistent utilities states that a single set of weights ( $w_{jx}$ ,  $w_{jy}$ , and  $w_{jz}$ ) and utilities ( $u_{jx}$ ,  $u_{jy}$ , and  $u_{jz}$ ) is assigned to each gamble  $j$  and a single utility  $u_C$  to each amount (price), independent of the preference measure. Context-dependent utility mappings permit one to assign different weights or utilities to each gamble, depending on the preference elicitation method. However, psychologically, this prevents application of a consistent evaluation process. Practically, using consistent utility mappings provides a substantial reduction in model parameters—if preference among a set of gambles is obtained via  $n$  measures, then context-dependent utility mappings permit an  $n$ -fold increase in weight and value parameters.

*Model Parameters*

The binary choice model is based on the discriminability index,  $d$ , but this index is entirely derived from the weights and values of the gambles. Therefore, no new parameters are required to determine this index. The step size ( $\Delta$ ) was chosen to be sufficiently small to closely approximate the results produced by a continuum of values (Diederich & Busemeyer, 2003; see Appendix A for the exact step sizes and number of steps used in the current analyses). In other words, we chose the step size ( $\Delta$ ) to be sufficiently small so that further decreases (i.e., finer grain scales) produced no

meaningful changes in the predictions (less than .01 in choice probability or mean matched value). The only parameter for the choice process is the number of steps,  $k$ , needed to reach the threshold,  $\theta$ . This threshold indicates the amount of evidence that must be accumulated to warrant a decision and could be used to model characteristics of the task (e.g., importance) and/or the individual (e.g., impulsivity).

For the indifference model, we introduced one new parameter,  $r$ , which reflects the tendency to end deliberation with an indifference response when one enters a neutral preference state between options at a given moment. It is important to note at this point that the binary response model without indifference is used whenever the decision maker makes a binary choice between two gambles in the standard experimental tasks discussed. However, the indifference model has been formulated here specifically for inclusion in the matching process for prices, CEs, and PEs.

The SVM model requires no new free parameters, because of the theoretical assumptions of the model. The only new parameter in this model,  $C_0$ , is used to specify the initial candidate considered in the search for a response value. As mentioned, this is set to the middle of the range for CE and PE tasks and to the bottom and top of the range for WTP and WTA tasks, respectively.<sup>5</sup> The range of candidate values is determined by the stimuli (e.g., gamble outcomes). The number of candidate values,  $k$ , is chosen to be sufficiently large to closely approximate a continuum. The step size for the value search process becomes  $\Delta = (\text{value range})/k$  for each gamble, defining the associated candidate values.

Almost all of the work is done by the computational model, which appears more complex than standard models of decision making. In reality, the model does not require an abundance of parameters, and its micro-operations are quite transparent (although its global behavior is more complex). Furthermore, we can now make predictions for single-valued responses, including buying prices, selling prices, CEs, PEs, and matched outcome values (in addition to choice responses). We posit one psychological mechanism that operates on two connected levels as a comprehensive process model of these different responses. For all value response measures, we derive response probability distributions ( $\mathbf{R}$ ) that predict the likelihood that each of the candidate values will be reported. In the following section, this framework is successfully applied to the empirical results that have challenged decision theories for over 30 years.

Accounting for Preference Reversal Phenomena

The motivation for our theory of preference reversals is to retain a consistent utility representation across various measures of preference. First, we show that the SVM model reproduces the qualitative patterns generated across a wide range of phenomena, using a single set of parameters. Second, we compute the precise quantitative predictions from the SVM model for a specific but complex data set. The latter analysis is used to examine the explanatory

<sup>5</sup> In practice, the matching process is mathematically defined in the SVM model with an initial distribution,  $C_0$ , based on a binomial distribution, that specifies the likelihood that each value is considered first (Figure 4; Appendix A). For buying prices, this initial distribution has a mode near the lowest value and is skewed to the right; for selling prices, the mode is near the highest value and is skewed to the left; for all other measures, the distribution is symmetric around a mode in the middle of the range.

power of the SVM model compared with alternative models that permit changes in the utilities across measures.

### Methods and Parameters for Qualitative Applications

First we apply the SVM model to the major findings of published empirical results (qualitative applications). Before discussing this application, we first provide details and rationale for the methods used (see Appendix A for further details about the computations of the predictions).

In the qualitative applications, not only do we retain consistent utility mappings, but we restrict our weight and value parameters further. It is important to show very clearly that preference reversals can occur even if we adopt a utility mapping based on the classic expected utility model (von Neumann & Morgenstern, 1944). By initially adopting the classic expected utility model, we can explore more fully the importance of the response process as a sole source of preference reversals. To accomplish this, we use the following simple forms for the weights and utilities in the qualitative applications:

$$w_x = p_x w_y = p_y w_z = p_z, \quad (10)$$

and

$$u_x = x^\alpha, \quad (11)$$

where  $\alpha$  is a coefficient to capture risk aversion. In other words, we use the stated probabilities to determine the decision weights, and we use a power function to represent the utility of payoffs. Thus, only one parameter,  $\alpha$ , is used in deriving the utility of all gambles for use in Equation 6, which is fixed throughout, so it is not even a free parameter. This formulation assigns a single utility value to each gamble, regardless of the response method.

Note that we do not endorse the expected utility model as an adequate model of preference, because it fails to explain well-known phenomena such as the Allais paradox (Allais, 1953; Kahneman & Tversky, 1979) as well as other phenomena (cf. Birnbaum, 2004). However, it is important to demonstrate that a complex utility model is not necessary to explain preference reversals. Of course, these more complex models also could be used to reproduce the same results.<sup>6</sup> For the qualitative applications, we selected  $\alpha = .70$  for the utility function,  $\theta = 3$  for the threshold bound, and  $r = .02$  for the exit rate. The same three parameters were used in all of the qualitative applications.

Unlike earlier deterministic models of preference, the SVM model predicts the entire probability distribution for choices, prices, and equivalence values. However, previous researchers have not reported the entire distributions but instead have given some type of summary statistic for each measure. Therefore, we must derive these summary statistics from the distributions predicted by our model. For the choice measure, we simply used the predicted probability that one gamble (Gamble L) will be chosen over another (Gamble H), as derived from our binary choice model. For all other measures, we computed the following summary statistics from the distributions predicted by SVM model. First, we computed the means and medians of the response value distributions for each gamble. Second, we computed the variance of the value distributions, which is an interesting statistic that has only rarely been reported in previous work. Finally, we computed a preferential pricing probability (PPP), which is defined as the

probability that a price (or PE) for Gamble L will exceed the price (or PE) for Gamble H, given that the values are different.<sup>7</sup> Thus, we can generate probabilistic preference relations for all response methods, which can be compared with reported frequencies in the literature. We should note, however, the distinction between the individual level of focus of our model and the aggregate data reported in empirical studies.

### SVM Application to Qualitative Empirical Findings

*Choice and pricing.* First, we examine within our framework the classic choice–pricing reversals between the representative L (35/36, \$4, \$0) and H (11/36, \$16, \$0) gambles presented earlier. These reversals typically entail choosing L in binary choice while assigning a higher price to H, and they are rather robust (e.g., Grether & Plott, 1979; Lichtenstein & Slovic, 1971; see Seidl, 2001, for a review). The SVM model can account for these reversals without changing weights, values, integration methods, or any of our model parameters between choice and pricing. Preference reversals are indeed emergent behavior of the deliberation process specified by the SVM model. To understand this behavior, we begin with choice–pricing reversals for which the pricing measure is the simple CE. Recall that, in this case, the SVM model begins search in the middle of the set of candidate values.

The predictions of the SVM model for the CE of each gamble are shown in the first row of Table 1, and the distributions are shown in Figure 5. The mean and median of the pricing distribution for H are greater than those for L, and PPP is less than .50, all of which indicate preference for H. However, the choice probability indicates preference for L, producing the classic choice–pricing reversal. Furthermore, the SVM model predictions show overpricing of H (i.e., the mean price exceeds the value that produces  $d = 0$ ) as the most significant factor, as supported by empirical studies (e.g., Tversky et al., 1990). It is important to note, however, that the SVM model does not predict reversals in all cases. That is, the model predicts reversals under circumstances that yield empirical reversals—and only under these conditions. For example, assume choice and pricing tasks involving the original Gamble H and another high variance gamble, H<sub>2</sub>, offering (13/36, \$10, \$0). In this case, the SVM model predicts that the choice probability (.61) will indicate preference for H, as does the probability (.72) that H will receive a higher price than H<sub>2</sub>.

<sup>6</sup> In fact, we have reproduced the qualitative results on preference reversals using a single configural-weight type of utility mapping, described later in the quantitative analysis section, for all measures.

<sup>7</sup> We did this as follows. Suppose we wished to find the probability that the price for Gamble L will exceed the price for Gamble H. First, for each possible response value of Gamble L (e.g., a price of \$X for Gamble L), we computed the joint probability that the value would occur and that the price for the other gamble would be lower (e.g., a price of \$X for Gamble L > \$Y for Gamble H). Then we integrated these joint probabilities across all candidate values \$X of Gamble L to obtain the total probability that the price for Gamble L would exceed the price for Gamble H. We then used a similar procedure to compute the probability that the price for Gamble H would exceed the price for Gamble L, for all \$Y. Ties were excluded from this calculation, so these probabilities do not sum to one. To normalize the probabilities, we divided each probability by the sum of the two probabilities.

Table 1  
*Sequential Value Matching Model Predictions of Preference Reversals Among Typical Gambles*

Variable	Pr[choose L]	PPP	Gamble L			Gamble H		
			\$ M	\$ Mdn	\$ Variance	\$ M	\$ Mdn	\$ Variance
Gains	.68	.27	3.42	3.60	0.31	4.82	4.80	4.13
Losses	.32	.73	-3.42	-3.60	0.31	-4.82	-4.80	4.13
Time discounting	.55	.47	4.08	3.00	0.87	5.75	4.00	3.50
Equal range	.69	.41	4.19	4.00	1.03	4.82	4.80	4.13
Equal variance	.98	.68	3.42	3.60	0.31	3.31	3.20	1.27
Buying prices		.93	52.07	52.20	2.80	37.65	37.20	104.58
Selling prices		.32	55.83	55.80	3.01	64.45	62.40	187.19

*Note.* The first five rows report model predictions for gambles similar to L and H reported in the text. The last two rows report results for gambles with two equiprobable outcomes and equal expected value (see text for all gambles). In all rows, a low-variance gamble (Gamble L) and a high-variance gamble (Gamble H) are compared, using one set of parameters in the sequential value matching model. Pr[choose L] > .50 and PPP > .50 indicate preference for Gamble L; thus, preference reversals occur within each of the first four rows and across the last two rows. Distributional measures in the row “Time discounting” have been divided by 100 to align with other values. Pr = probability; PPP = preferential pricing probability.

The SVM model also predicts choice–pricing reversals when gambles offer only losses, such as those created when we simply change the signs of the outcomes on L and H.<sup>8</sup> The SVM model predicts the opposite reversals in this case—preference for Gamble H in choice but preference for Gamble L when one is inferring from CEs. These results, shown in the second row of Table 1, are also consistent with empirical results (Ganzach, 1996). Thus, the SVM model explains reported preference reversal differences between the gain and loss domains without loss aversion or changing valuation.

Research has also shown preference reversals between choice and pricing when options offering a certain but delayed outcome are used (Tversky, et al., 1990; see also Stalmeier, Wakker, & Bezembinder, 1997). Specifically, consider an Investment L (offering a return of \$2,000 after 5 years) and an Investment H (\$4,000 in 10 years), from Tversky et al. (1990). To apply the SVM model, we maintain the same form for determining  $u_x$  using  $\alpha = .7$  for consistency. To convert the time delay into an outcome weight, we use simple (and parameter-free) hyperbolic discounting:  $w_x = 1/(1 + \text{delay})$ . The results, in the third row of Table 1, again support the reported empirical trend (Tversky et al., 1990): choice of the smaller investment return received sooner (L), but a higher price attached to the larger investment received later (H).

It does not appear that the SVM model predictions are tied to specific types or numbers of gamble outcomes, as shown by the analyses so far. What property is it, then, that the SVM model is using to correctly predict all of these results? Perhaps the SVM explanation relies on the smaller range of L compared with H—that is, higher pricing of H may be due simply to the greater range of candidate values from which to select. We can change the L and H gambles slightly to examine this by equating their ranges. Adding a third outcome to Gamble L—with the same value as the win in Gamble H but with a nominal probability—results in identical candidate values for each gamble, without greatly affecting the original gamble properties.<sup>9</sup> Yet, even when the range of candidate values is the same, the classic reversal pattern is still obtained (see Row 4 in Table 1). This prediction is supported empirically for gambles with equal ranges (Busemeyer & Goldstein, 1992; Jessup, Johnson, & Busemeyer, 2004).

The crucial stimulus property that generates the preference reversals in the SVM model is not the range of outcomes but the variance of the gambles. The variance for a single gamble can be

thought of as a measure of uncertainty about its value. As this outcome uncertainty increases, it leads to greater fluctuation in the momentary evaluation of a gamble. Consequently, the distribution of prices should reflect this uncertainty, suggesting a positive correlation between the gamble variance and the response variance (and this has indeed been found in the data reported by Bostic, Herrnstein, & Luce, 1990). This relation can be seen in Figure 5, where there is greater variance in the CE distribution for the high-variance Gamble H compared with Gamble L. If one thinks of the SVM model as a search for the “true”  $C^*$ , then increasing the variance decreases the ability to discern the true price and thus decreases the likelihood of finding it. Thus, as the variance of a gamble decreases, we expect less variance in the response prices, which will allow convergence toward a better estimate of the true  $C^*$ .

Consider next the variance in a pair of gambles, which can be conceptualized as the ability to discriminate between the gambles. Even if we assume equal variance for each of two gambles, if the total variance is small, they should be easier to discriminate, and thus the choice probabilities will be more extreme. To check these predictions, we artificially removed the high variance from Gamble H in the mathematical SVM formulas (decreasing  $\sigma_H$  to  $\sigma_L$ ), without actually changing the input stimuli at all (no change in  $d$ ). Indeed, the response variance decreased around the true utility value for Gamble H (i.e., the value producing  $d = 0$ ), the ease of discrimination made the choice probability extreme, and the preference reversals disappeared (fifth row of Table 1).

Sensitivity analyses of the free parameters confirm that the payoff variance is the primary impetus for the preference reversals, although other parameters may interact. Increases in the exit rate,  $r$ , lead to changes in the price for the high-variance gamble (Gamble H), with relatively little change in the reported price for

<sup>8</sup> The SVM model can also account for choice–pricing reversals when mixed gambles—those offering both gains and losses—are used. The analysis is the same as for gambles offering only gains and has therefore been excluded here for the sake of brevity.

<sup>9</sup> The new Gamble L becomes, specifically,  $\$x = 4$ , with  $p_x = 35/36 - 1/1,000$ ,  $\$y = \$0$  with  $p_y = 1/36$ , and  $\$z = \$16$  with  $p_z = 1/1,000$ , yielding an expected value only 1.2¢ greater than the original Gamble L.

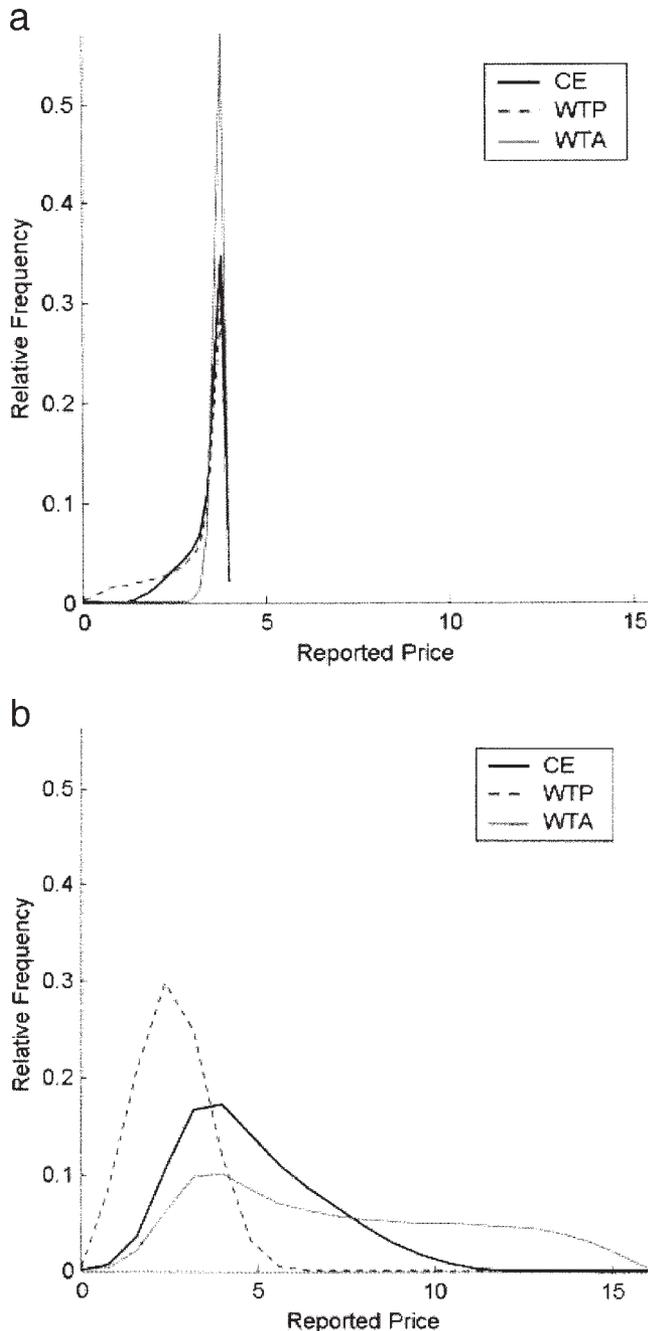


Figure 5. Sequential value matching model predictions for certainty equivalent (CE), willingness to pay (WTP), and willingness to accept (WTA) distributions of (a) Gamble L (35/36, \$4, \$0) and (b) Gamble H (11/36, \$16, \$0).

the low-variance gamble (Gamble L). This is in accord with the empirical findings by Bostic et al. (1990) and consistent with the explanation for the effect of gamble variance—as the probability of exiting increases, the value matching has a lower chance of reaching the true price (price that produces  $d = 0$ ), especially for H.

We can similarly explore changes in the start of the value search, the initial state,  $C_0$  (see Figures 4 and 5). Again, the greater impact is on H, with larger (compared with L) increases

in the mean price as the initial candidate increases. This explains attenuation in the incidence of choice–pricing reversals when buying prices are used (Ganzach, 1996; Lichtenstein & Slovic, 1971), because the SVM model assumes initial values skewed toward the lower end of the candidate range for this measure. In fact, with our process specification of the initial candidate values for buying prices, the SVM model could even reproduce the challenging results reported by Casey (1991). In particular, he found that large outcomes and the use of WTP, as opposed to commonly used CE and WTA measures, can produce higher pricing of the low-variance Gamble L, compared with H. To explore this result in the SVM model, we begin by using representative two-outcome gambles, L (0.98, \$97, \$0) and H (0.43, \$256, \$0), from Casey (1991). In this case, using the same parameters as in previous applications, we obtain  $PPP = .76$ , which equals the marginal probability reported in Casey (1991), to the second decimal place, of higher buying prices on L than H.<sup>10</sup> The SVM model is thus able to account for the effects of different pricing methods on the classic preference reversal between choice and price; now we see whether our specification also predicts preference reversals between different pricing methods.

*Buying and selling prices.* For within-pricing reversals, assume Gamble L offers (.5, \$60, \$48) and Gamble H offers (.5, \$96, \$12). Birnbaum and colleagues (e.g., Birnbaum & Beeghley, 1997; Birnbaum et al., 1992; Birnbaum & Zimmermann, 1998) have used these and similar gambles—with equiprobable outcomes and equal expected values but different variances—and found that they produced predictable within-pricing reversals. Typically, a higher WTP is given to L, whereas the WTA is greater for H. The SVM model can account for the pricing reversals using the process specification in the previous section and the same  $\alpha$ ,  $r$ , and  $\theta$  parameters used to predict the choice–pricing reversals above. Specifically, the model suggests preference for L ( $PPP > .50$ ) when it predicts buying prices but preference for H ( $PPP < .50$ ) when it predicts selling prices (Table 1, last two rows). Our specification of the initial candidate distribution to mimic competitive pricing is largely responsible for this result (cf. Figure 4). However, the variance of the gambles still plays an important role.

As before, increased stimulus (gamble) variance generates decreased discriminability, causing larger fluctuation in the reported price—that is, a positive correlation between gamble variance and response variance. Depending on the direction of initial bias, or skew, induced by the role of the agent (buyer vs. seller), the decreased discriminability leads to a higher probability of reporting values further from (less or greater than) the true indifference price that causes  $d = 0$ . This affects the high-variance Gamble H more than L, because of the associated increase in response variance, independent of the skew direction. We can once again examine this prediction by equating the variance in the SVM

<sup>10</sup> Casey (1991) also found instances of “reverse reversals,” where higher pricing of L accompanied choice of H. To account for choice of the high-variance gamble, we need to depart from our use of a single parameter set in one respect. Specifically, we must decrease the risk aversion, which corresponds to increasing the utility exponent slightly,  $\alpha = .90$ . This results in choice probabilities favoring Gamble H,  $\text{Pr}[\text{choose H}] = .56$ , but retains higher pricing for Gamble L,  $PPP = .62$ . Note that this still retains converging operations by holding parameters constant across elicitation methods, if not across experimental samples.

equations. In this case, the reported prices of H converge toward the true value for both WTP and WTA, overcoming the initial bias and eliminating the preference reversals.

*CEs and PEs.* Hershey and Schoemaker (1985) conducted direct tests of consistency between preference relations inferred from CE and PE measures of preference. They found that the majority of participants produced systematic inconsistencies between these measures. An earlier application of the SVM model (a special case of the current model) was shown to be able to account for these results without assuming any changes in utilities across CE and PE methods of valuation (see Townsend & Busemeyer, 1995, for details). Here we examine a more challenging case that includes CE and PE measures of preference, as well as choice and competitive pricing.

*Multiple measures.* Each previous theory of preference reversals can explain some of the reversals between pairs of methods covered so far, yet none of them can account for all the phenomena. Here, we examine the ability of the SVM model to simultaneously account for responses from studies using multiple (three or four) preference measures. In particular, we first apply the SVM model to data from Slovic et al. (1990), which elicited preference for a set of 16 gamble pairs using choice, selling prices, PEs, and matched payoffs. Whereas the PE task involves filling in a missing probability to cause indifference, a payoff matching task involves filling in a missing payoff for a gamble. Once again, we do not adjust any parameters between response modes, but instead we apply the SVM model to the entire set of Slovic et al. (1990) stimuli using the same parameters used in previous analyses.

The predictions of the SVM model and the summary data from Slovic et al. (1990) are shown in Table 2, indicating the success of the SVM model in simultaneously predicting preference relations for multiple measures. In Table 2, the PPPs for PEs and matched payoffs were computed in relation to the missing value.<sup>11</sup> For example, we calculate PPP for matched payoffs by (a) computing the probability of a response value greater than the missing value in Gamble L, (b) computing the probability of a matched response value less than the omitted value in Gamble L, and (c) dividing the second probability by the sum of the two. Because a matched response value greater than the value omitted from Gamble L indicates preference for

Gamble H and vice versa, the PPP has the same preference implications as with the other elicitation methods. The predictions replicate the empirical trend: For both methods where responses are given in dollar values (matched payoff and WTA), preference for H is predicted ( $PPP < .50$ ), whereas the other two measures (PEs and choice) indicate preference for L. We thus account for preference rankings across four measures in one study, with one consistent set of parameters.

Another study involving multiple measures was reported by Ganzach (1996). This experiment included preference measures based on choices, buying prices, and selling prices. Recall that the main finding was that selling prices lead to greater preference for the high-variance gamble in a pair, but choice and buying prices show greater preference for the low-variance gamble. Furthermore, this study used gambles that had five equiprobable outcomes. Thus, not only can we examine these three key response modes simultaneously, we can also further generalize the success of the SVM model to five outcomes. As can be seen in Table 3, the SVM model predicts the correct qualitative pattern of results.<sup>12</sup> The high-variance gamble is preferred when we use WTA, but choice and WTP reveal preference for the low-variance gamble. The results in Table 3 were computed with the stimuli and data from Experiment 2 in Ganzach (1996), and the SVM model predictions were generated again from the same parameters as in all previous applications.

*Process tracing.* Some studies examining preference reversals have included process-tracing measures that may help to further support the proposed process of the SVM model. A study by Schkade and Johnson (1989) provided the best process-tracing data for reversals between choice and pricing (here, WTA) measures; the results indeed coincide with the qualitative predictions of the SVM process. First, the authors observed the equivalent of an initial candidate value by recording at what point along the pricing response scale participants first positioned a computer mouse. They found that the initial candidate value for the high-variance gamble was greater than that for the low-variance gamble, which is consistent with the SVM process applied to these gambles (because of the greater outcome of the high-variance gamble). Second, the initial candidate value for the high-variance gamble was greater in those cases in which a reversal occurred, compared with instances in which no reversal occurred. The SVM model would also correctly predict that decreasing the initial candidate value would lead to decreases in the incidence of reversals (cf. Figure 4). Finally, initial candidates showed the highest correlation (among all gamble elements) with a gamble's highest outcome, consistent with the SVM specification for WTA.

Table 2  
*SVM Model Predictions and Empirical Data for Gambles in Slovic et al. (1990)*

Response method	Reported frequency	SVM model predictions
Choice	.76	.64
PE	.73	.89
Matched payoff	.49	.38
WTA	.37	.20

*Note.* Reported frequency is defined as the proportion of participants who indicated preference for Gamble L for each response method, reported in Slovic et al. (1990). Sequential value matching (SVM) model predictions are the probability of choosing Gamble L in the first row and the preferential pricing probability in all other rows, where values greater than .50 always indicate preference for Gamble L. The correlation between reported frequencies and SVM measures is .92. PE = probability equivalent; WTA = willingness to accept, or selling price.

<sup>11</sup> We derive PE predictions by using a missing probability in each gamble, then averaging the resulting PPPs. For payoff matching, we derive predictions only for the case in which the missing payoff value is in Gamble L. In this case, it is reasonable to assume the upper limit of the value range is set equal to the value that is presented to participants (i.e., the outcome of the displayed Gamble H) and the lower limit is set to zero. However, we can also reproduce the results when the missing payoff is in Gamble H, provided we make a reasonable estimate for the upper bound for the value range for this case.

<sup>12</sup> Ganzach (1996) reported proportion of preference for the high-variance gamble (PHVG); we use the conversion  $PPP = 1 - PHVG$  for ease of comparison in Table 3.

Table 3  
*Sequential Value Matching Model Predictions and Empirical Data for Gambles in Ganzach (1996)*

Response method	Reported PPP*	Predicted PPP
Choice	.68	.54
WTP	.57	.67
WTA	.41	.39

*Note.* Reported PPP\* is determined by  $1 - \text{PHVG}$ , where PHVG is the proportion of responses indicating preference for the high-variance gamble reported in Ganzach (1996). PPP = preferential pricing probability; WTP = willingness to pay; WTA = willingness to accept.

*Response times.* Schkade and Johnson (1989) also reported the only response time data of which we are aware in preference reversal research. The dynamic nature of the SVM model allows for response time predictions that other models cannot produce. First, Schkade and Johnson (1989) found that the response time in choice was significantly less than that in pricing. This may seem counterintuitive if choice involves separate evaluation of two options, compared with the single evaluation necessary to price an option. Because the SVM pricing process consists of multiple implicit choice processes (i.e., repeated use of the comparison layer), the model predicts this basic result. Second, it was found that the time required to price a high-variance gamble was significantly greater than that for pricing a low-variance gamble. The lower discriminability of the high-variance gamble leads to longer latencies for comparison layer outputs and thus longer pricing response times, relative to the low-variance gamble, producing the correct prediction.<sup>13</sup> Previous work has also shown the ability of the SVM choice component (DFT) to account for detailed results such as speed-accuracy trade-offs (Busemeyer & Townsend, 1993) and various effects of time pressure (e.g., Diederich, 2003).

*Distributional predictions.* The SVM model makes strong, testable predictions regarding the response (pricing) distributions that are not possible with competing deterministic approaches. In particular, the SVM model makes predictions regarding the relative variance of the pricing distributions for high- and low-variance gambles as well as predictions concerning skew for WTP versus WTA. As mentioned earlier, the SVM model predicts that the response variance will be directly related to the gamble variance, causing greater variance in the pricing distribution for high-variance (as compared with low-variance) gambles. This prediction is supported by empirical findings from Bostic et al. (1990) and Jessup et al. (2004). The SVM model also predicts that the skew in the initial candidate values for WTP and WTA will be evident in the response distributions. Specifically, this suggests that the response distributions will have a positive skew for WTP responses and a negative skew for WTA responses. Initial support for this prediction comes from Jessup et al. (2004), who found greater skew for WTP compared with WTA for the "standard" L and H gambles from Lichtenstein and Slovic (1971).

In summary, we have shown in these qualitative applications how the SVM model can account for key preference reversal phenomena reported to date (see Table 1); this includes process data, response time data, and distributional predictions that are not possible with other theories (see Figure 4). The remarkable success of the SVM model is even more impressive when we consider that

the free parameters, weighting methods, and utility representations were held constant across all analyses. Consequently, the SVM model enjoys success in applications to multiple measures as well (Tables 2 and 3). The SVM model can make remarkably good qualitative predictions, but one can still question the quantitative accuracy of these predictions. This issue is addressed in the next section.

#### *Quantitative Application of SVM Model to Empirical Data*

We have shown that the SVM model correctly predicts a wide variety of qualitative findings for various preference reversals among six different elicitation methods. In this section we focus on deriving more precise predictions for two preference measurement methods, WTP and WTA, across a large set of gambles. Furthermore, we directly compare the predictive ability of the SVM model, which assumes stable utilities and dependent response mappings, with a configural weighting model assuming the converse.

For the quantitative analyses in this section, we used the data from a large study by Birnbaum and Beeghley (1997).<sup>14</sup> This study elicited WTP and WTA for 168 gambles with three equiprobable outcomes. In particular, the gambles consisted of values for the first two outcomes that produced 10 different ranges, crossed with six values of the third outcome (see Birnbaum & Beeghley, 1997, for design details). Birnbaum and Beeghley (1997) found that (a) WTA was greater than WTP for all gambles, (b) increasing the third outcome produced nonlinear increases in both WTP and WTA, (c) responses violated branch independence (the value of the third outcome interacted with the range of the other two outcomes in determining prices), and (d) violations of branch independence occurred for more (lower) values of the third outcome for WTA compared with WTP. This pattern of results also leads to various preference reversals across pricing methods as well as common outcomes (see Figure 6 for the actual data).

The primary purposes of this section are to show that the SVM model can account for these detailed trends and to compare it with the configural weight model used by Birnbaum and Beeghley (1997). First, we describe exactly how each model is formulated. Then, we discuss the SVM model predictions in relation to the main empirical trends. Finally, we compare the predictive accuracy of the competing models using the proportion of explained variance in the actual responses.

There were slight changes to the specific assumptions used in the quantitative application, as compared with the assumptions used in the qualitative application. These changes were made to match the assumptions used in the original analysis by Birnbaum and Beeghley (1997). In other words, we wished to equate the SVM model with the Birnbaum and Beeghley models in all re-

<sup>13</sup> We use the same assumptions and parameter values as in all other applications and also assume that a unitary time interval is required for each step in the preference state (comparison layer) and candidate value increment (matching layer). Therefore, the mean number of total steps required for reporting WTA for Gamble L in Figure 5 is 51.59, and for Gamble H in Figure 5 it is 54.86 (see Johnson, 2004; Shiffrin & Thompson, 1988, for details).

<sup>14</sup> These data were recommended to us by Michael Birnbaum as a challenge to our theory. We thank him for providing the detailed data.

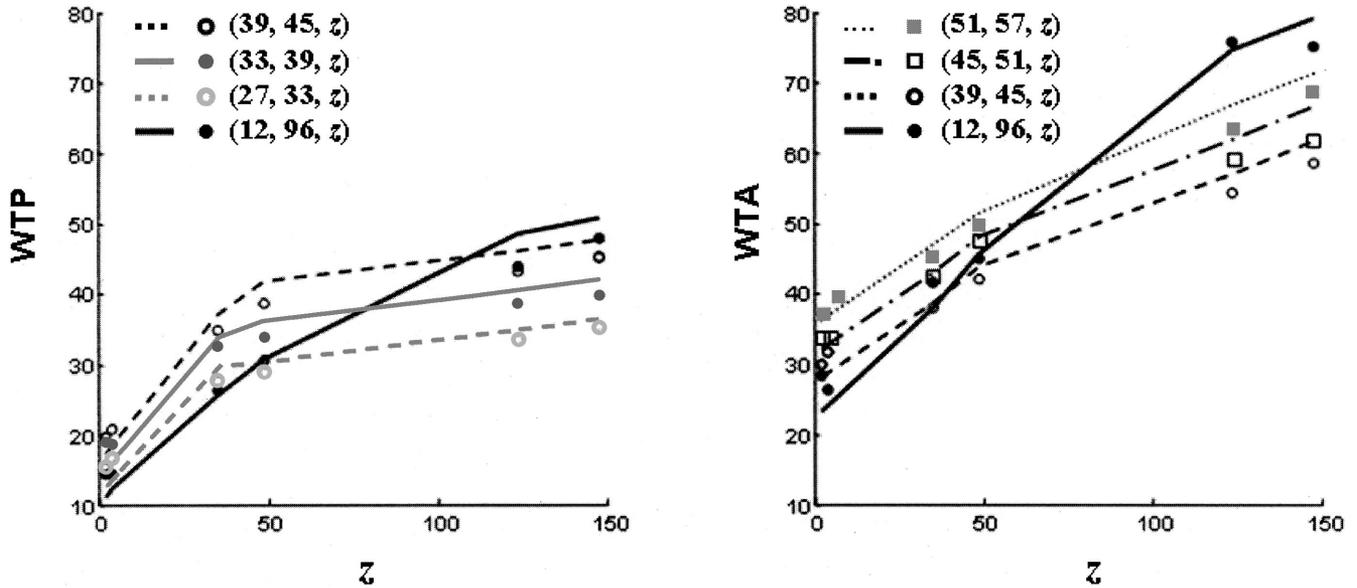


Figure 6. Sequential value matching (SVM) model predictions and empirical data for gambles in Birnbaum and Beeghley (1997). The horizontal axis indicates the value of the common outcome between gambles, and the vertical axis indicates the mean price (willingness to pay [WTP], left, or willingness to accept [WTA], right). Points represent mean prices from the original data. Lines show best fitting SVM model predictions (see Birnbaum & Beeghley, 1997, for gamble outcomes). Gambles offered three equiprobable outcomes, shown in parentheses. See text for interpretations.

spects except the critical properties that we wished to test. First, Birnbaum and Beeghley (1997) fitted different versions of their model, all of which allowed the weights to be free parameters; therefore, we also allowed the weights in the SVM model to be free parameters. The key distinction is that the SVM model uses the same weights across response methods (a consistent utility mapping), unlike the models in Birnbaum and Beeghley (1997). Second, the set of models examined by Birnbaum and Beeghley (1997) included a power function transformation from dollars to utilities, which added an extra parameter exponent,  $\alpha$ . However, Birnbaum and Beeghley (1997) found that for the small range of payoffs used in their study, setting  $\alpha = 1$  produced adequate fits. Therefore, we followed Birnbaum and Beeghley (1997) and used the version of their utility model that assumed  $\alpha = 1$ .

The quantitative application is characterized by other minor changes. Only the mean prices were reported and fitted by Birnbaum and Beeghley (1997). Therefore, we used the means of the SVM response pricing distributions rather than the PPP measure derived for the qualitative analyses. Finally, the exit rate,  $r$ , was not preset as in the qualitative applications but was allowed to be fitted to the data as a free parameter. All other parameters and assumptions were the same as in the qualitative applications.

**Model descriptions.** Birnbaum and Beeghley (1997) proposed a configural weight utility model of prices for gambles. According to this model, the price for a three-outcome  $(x_L, x_M, x_H)$  gamble with equal probabilities is given by

$$\$X = u^{-1}[w_L \times u(x_L) + w_M \times u(x_M) + w_H \times u(x_H)], \quad (12)$$

where  $w_j$  is called the configural weight for outcome  $j$  and these weights are assumed to vary depending on the rank order (lowest, middle, or highest) of the outcomes within a gamble. The weights

sum to 1.0, so  $w_H = (1 - w_L - w_M)$ . Given the limited range of positive payoffs used in this study, the utility function  $u(x)$  was found to be well approximated by a linear function so Birnbaum and Beeghley simply assumed  $u(x) = a \times x + b$ ; this implies, of course, that  $u(x)^{-1} = x/a - b$ , so that

$$\$X = w_L \times x_L + w_M \times x_M + w_H \times x_H. \quad (13)$$

A key assumption of the Birnbaum and Beeghley (1997) model is that the weights in Equation 13 are free to change across WTA and WTP measures.

The SVM model was based on these same assumptions: (a) The utility function was assumed to be linear,  $u(x) = x$ , and (b) we estimated two attention weights for the low and medium outcomes,  $w_L$  and  $w_M$  (the attention weight to the highest outcome was then fixed equal to  $1 - w_L - w_M$ ). Together, the two assumptions imply that Equation 13 was used to derive  $E[V_j(t)]$  for any gamble  $j$  and  $E[V_C(t)]$  was used for certain cash values. Unlike Birnbaum and Beeghley (1997), we used a common set of attention weights for WTA and WTP measures. Finally, the exit rate,  $r$ , is a free parameter. We then computed the predictions for the SVM model using the mean of the distribution generated by the equations introduced earlier and detailed in Appendix A.

Altogether, we compared four models, presented next in order of complexity. The first is a configural weight model (CW1) that retains a single utility mapping across response methods; this model has two free parameters ( $w_L, w_M$ ). The second model is the SVM model, which has three free parameters ( $w_L, w_M, r$ ). The third model is a configural weight model (CW2), endorsed by Birnbaum and Beeghley (1997), that allows for different utility mappings and thus has four free parameters ( $w_{L,WTP}; w_{M,WTP}; w_{L,WTA}; w_{M,WTA}$ ). The final model is based on the SVM model,

but without consistent utilities; this also allows for different weights across buying and selling prices, resulting in five free parameters ( $w_{L,WTP}$ ;  $w_{M,WTP}$ ;  $w_{L,WTA}$ ;  $w_{M,WTA}$ ;  $r$ ). We fitted each model to the mean WTP and WTA (averaged across the 46 participants) by optimizing the free parameters to minimize the summed squared residuals between data and model predictions for all 168 gambles from Birnbaum and Beeghley (1997).

*SVM model predictions.* Birnbaum and Beeghley (1997) showed that CW1 fails to account for the data, whereas the CW2 model provides a very good fit to the results. First, we show that the SVM model also provides a very good account for the four key results mentioned earlier. Figure 6 plots the mean of the SVM-predicted response distribution, separately for a representative subset of 24 buying prices and a different subset of 24 selling prices (cf. Birnbaum & Beeghley, Figure 1, p. 89, and Figure 2, p. 90, respectively); the figure also shows the actual data for comparison. The SVM model predicts (a) that WTA will be greater than WTP for all gambles and (b) nonlinear increases in WTP and WTA as a function of the third outcome, in accord with the data (see Figure 6). Furthermore, the model reproduces (c) the interaction of range and third outcome that produces violations of branch independence, as can be seen by the intersections of lines in Figure 6. That is, whether the high- or low-variance gamble is preferred depends on the value of the common outcome. Finally, the SVM model correctly predicts (d) the violations of branch independence first appear for lower third-outcome values for WTA versus WTP, illustrated by the earlier (along the  $x$  axis) intersection of the dark solid line and the dark dashed line in Figure 6.

Intuitively, adding a large third outcome to both a high- and a low-variance gamble will have a greater effect on the low-variance gamble, reducing range discrepancies between the two gambles. In contrast, a small third outcome has little effect on the variance, especially that of the lower variance gamble, allowing easier discrimination for the latter. The combined results also suggest some preference reversals between the lower variance and higher variance gambles, as we can see by pairing the mean prices for WTP and WTA in Figure 6. For example, the dark dashed line represents the same gamble in both the left and right panels of Figure 6; however, the dark solid line (also representing a common gamble) crosses the latter much sooner than the former—producing areas where WTA for the solid line is greater but WTP for the dashed line is greater. That is, for the associated gambles in the figure (and wherever else such patterns occur), there is a preference reversal depending on whether WTP or WTA is used to infer relations.

The best fitting SVM model parameters producing this result were weights of .45, .51, and .04 for the lowest, middle, and highest outcomes of the gambles, respectively, and an exit rate of  $r = .02$ . Note that this best fitting value of the exit rate is similar to the value used in the qualitative results (equal to two decimal places), which provides further encouraging support for those results. For comparison, the weights for the best fitting CW2 model were, for buying prices, .57, .38, and .05, respectively, and, for selling prices, .32, .51, and .17, respectively.

*Model comparisons.* It is evident that the SVM model provides a good account of the data—but how does it compare with other explanations? To answer this, we computed the proportion of explained variance for each of the four models. The CW1 model predicts the same deterministic response values for buying and selling prices, because it only uses one set of weights and does not

specify a response bias; this provides a poor fit to the data ( $R^2 = .78$ ). The SVM model also suggests one set of weights for both buying and selling prices but specifies a stochastic response process that is able to account remarkably for the data ( $R^2 = .96$ ). The CW2 model does not take advantage of this response specification but is able to account equally well for the data ( $R^2 = .96$ ) by alternatively assuming differential weighting across buying and selling contexts. Thus, the response process of the SVM model can provide an equally good fit to the data as approaches that assume contingent weighting, while retaining a consistent evaluative mechanism.<sup>15</sup> In fact, allowing contingent weighting instead of (in addition to) the response process of the SVM model increases the fit only beyond the second decimal place, at the cost of one (two) additional parameter(s).

We have shown in this section how the SVM model predicts detailed trends in empirical data, such as the violations of branch independence in Birnbaum and Beeghley (1997). Furthermore, we have shown how our approach—assuming context effects in the response process rather than in the evaluation of weights and values—can provide an equally good fit to the data as a configural weight model (CW2). In addition, the SVM model uses one less parameter than the CW2 model for the data set fitted here in achieving this performance. Finally, it is important to note that as the number of outcomes and/or response methods increases, the difference in the number of free parameters between the SVM model and the CW2 becomes even larger. For example, to fit the data from Ganzach (1996) in the same manner, the SVM model would require only two additional parameters (five total), whereas the CW2 model would require an additional eight parameters (12 total).

### *Comparing the SVM Model With Other Theories*

The qualitative and quantitative applications of the SVM model indicate that it provides a comprehensive account for preference reversal phenomena. The success of the SVM model is coupled with the parsimony of a consistent mechanism for preference responses that operates on a single, stable utility structure. Although ours is one of many explanations that have been offered to account for preference reversals across elicitation methods, we believe the current theory holds distinct advantages when we consider all the phenomena herein.

*Contingent weighting.* Tversky and colleagues (Slovic et al., 1990; Tversky et al., 1988) explained many reversals by contingent weighting—specifying psychologically based assumptions about changes in the relative weight given to probability or value information. First, they assumed a prominence effect, such that the more prominent dimension is weighted more heavily in choice than in other tasks. The more prominent, or salient, dimension in

<sup>15</sup> The same picture emerges when we compare squared correlations of predicted prices with empirical data, using both the Pearson correlation coefficient and the Spearman rank-order correlation coefficient. We can also consider correlations to changes in rank order, which provide a more precise test (Birnbaum & Beeghley, 1997). In this case, the SVM model achieves the same correlation (.94) as the CW2 when we consider changes in rank order across response methods and achieves a higher median correlation (.86 vs. .78) to changes in rank order across values of the third outcome (violations of branch independence).

choice is assumed to be the probability information, in accord with empirical results (Tversky et al., 1988). Second, the compatibility hypothesis states that the dimension that is more compatible with the response is weighted more heavily in the associated task. This results in greater weight for value information in pricing tasks and greater weight for probability information in the PE task. In conjunction, these two hypotheses are able to explain reversals between choice and pricing and between PE and CE methods. However, this theory cannot explain why reversals are less prevalent when choice is paired with buying prices (rather than selling prices), because the compatibility and prominence effects should operate in the same way for both pricing measures. More important and for the same reason, this account cannot explain the reversals between WTP and WTA. Furthermore, contingent weighting cannot explain choice–pricing reversals among gambles with equally probable outcomes (e.g., Ganzach, 1996), because increased weight on probability information does not favor either gamble in this case. In contrast, the SVM model predicts differences in WTP and WTA as a result of the initial candidate values considered. Because we do not rely on differential weighting of probabilities, our approach is also able to explain preference reversals among gambles with equally likely outcomes.

*Contingent valuation.* An alternative account for the discrepancies between WTP and WTA is to change the evaluation of outcomes (i.e., the utility function) across these two measures (Tversky & Kahneman, 1992). In particular, one may assume that the discrepancies between WTP and WTA are the result of different reference points, or “endowment effects” and aversion to losses (Kahneman et al., 1990; Tversky & Kahneman, 1991, 1992). According to this idea, one may evaluate gamble outcomes as well as the gamble price differently, as gains or losses, depending on the response mode. For example, if one is stating a buying price, perhaps one views the price paid as a loss whereas the gamble outcomes are considered gains; alternatively, a selling price may be seen as a gain, whereas the possible gamble outcomes are given up (a loss) in selling. If we assume such reference point effects and loss aversion, evaluations could change, causing discrepancies between different pricing methods. However, although such hypotheses may produce discrepancies between WTP and WTA responses for a gamble, these hypotheses are unable to account for reversals between WTP and WTA (see Appendix B, and Birnbaum & Zimmermann, 1998, Appendix B). Finally, changing outcome evaluations across response modes does not account for reversals between CE and PE.

*Contingent operations.* Several researchers have proposed *change of process* theories of preference reversals, which do not require changing weights or values (Mellers, Chang, et al., 1992; Mellers, Ordoñez, & Birnbaum, 1992; Payne, Bettman, & Johnson, 1993). For example, Mellers and colleagues have explained preference reversals between attractiveness ratings and selling prices by assuming that an additive rule is used to combine probability weights with values for attraction ratings but a multiplicative rule is used to combine probabilities and values for prices (Mellers, Chang, et al., 1992; for other contingent processing accounts, see Payne et al., 1992). However, this theory explicitly assumes the same combination rule for buying and selling prices, so this theory cannot explain preference reversals between these two measures. Multiplicative rules are also commonly used to explain choices between risky gambles (see Luce, 2000, for a review), which makes it difficult for the contingent operations

model to explain preference reversals between choice and prices without changing the weights and values across measures.

*Intransitive preferences.* Loomes and Sugden (1983; Loomes, Starmer, & Sugden, 1989) suggested that intransitive preferences can result in revealed preference reversals in accord with their *regret theory* (Loomes & Sugden, 1982; see also Bell, 1982, and Fishburn, 1982, for similar theories). These theories posit consideration of the anticipated elation (or regret) experienced from avoiding a loss (or foregoing a gain) as distinct components in the utility equation. This addition can account for a variety of phenomena, including reversals between prices and choices (Loomes & Sugden, 1983). However, Loomes and Sugden’s theory has not been shown to account for reversals between WTP and WTA, and their explanation for choice–pricing reversals seems to apply identically for all measures. Furthermore, their theory requires additional assumptions on the forms of valuation functions, which then determine specific relations among gamble elements that must be met for choice–pricing reversals to occur. Finally, this theory also has not been shown to account for reversals between CE and PE, and it is not readily apparent how this theory would do so.

*Anchoring and adjustment.* Anchoring and adjustment models by Lichtenstein and Slovic (1971), Goldstein and Einhorn (1987), and others can explain preference reversals between choice and pricing by assuming insufficient adjustment of an anchor value in pricing. However, these theories do not have a specific mechanism for the amount of adjustment. Consequently, the predictions of anchoring and adjustment theories depend on using gambles with different ranges of outcomes, thus producing different anchors. Given gambles with equal ranges of outcome values, anchoring and adjustment should not produce differences in rank order across preference measures. The empirical finding of preference reversals when outcome ranges are equal across gambles (Busemeyer & Goldstein, 1992; Jessup et al., 2004) provides evidence against these theories. The SVM model produces this result naturally because of the model’s dependence on gamble variance rather than outcome range. Second, although anchoring and adjustment theories describe a process, they are not typically formulated as such. That is, anchoring and adjustment theories can be formalized as an averaging model, which then reduces anchoring and adjustment to differential weighting (e.g., Birnbaum & Zimmermann, 1998, Appendix E; Hershey & Schoemaker, 1985). In short, this means that earlier anchoring and adjustment models result in giving more weight to the anchor outcomes, whereas the SVM model simply uses the anchors as a starting position for a mathematically specified dynamic adjustment mechanism. In this sense, perhaps the computational model presented here is more in the spirit of anchoring and adjustment than an algebraic differential weighting formulation.

*Configural weighting.* Perhaps the most flexible approach has been adopted by Birnbaum and colleagues (Birnbaum et al., 1992; Birnbaum & Sutton, 1992), and it is capable of accounting for reversals between WTP and WTA as well as for the “original” reversals between choice and pricing measures. Birnbaum et al. proposed a configural weighting mechanism, in which the weight given to each gamble outcome is a function of its probability as well as its rank (among all outcomes). One can use this approach to fit empirical data from preference reversal studies by adjusting the parameters of the weighting function separately for each measurement method. That is, a different weighting function could be applied to each of choice, WTP, and WTA. This is a different

pursuit, methodologically, than specifying applicable mechanisms beforehand, as in the SVM model. The increased flexibility of configural weighting comes at the expense of an increase in parameters over the SVM model. The benefit of fitting weights is that configural weighting can potentially account for many of the reversal phenomena. However, this approach has not been applied to PE methods and thus cannot a priori predict reversals between CE and PE measures. Also, it is unclear how configural weight would be “transferred” when there is only a single outcome, so the application of this approach to the delayed investment example is also unclear.

A final major advantage of the SVM model over all of the approaches above is the ability to explain novel results, such as distributional properties of the reported prices. The other approaches are static and deterministic, lacking mechanisms for explaining the systematic relation between the variance of a gamble and the corresponding variances of the prices. The SVM model automatically generates the correct predictions for this basic property as well as predictions for the skew of the distributions for buying and selling prices. Also, the dynamic mechanisms of the SVM model provide precise numeric predictions of response times for choice and pricing, which other theories do not. These SVM model predictions are also in accord with published data on response times for choice versus pricing and for pricing of high-versus low-variance gambles (Schkade & Johnson, 1989). Finally, because the SVM model is formulated as a process model, it also makes specific predictions that can be tested via process-tracing techniques. Although such procedures have rarely been used in preference reversal research, the available data support our hypothesized process (Schkade & Johnson, 1989).

Previous attempts to explain preference reversal phenomena have met with only limited success. In contrast, we developed the SVM model by considering the collective results of decades of research and formulating a coherent theory for all of the elicitation methods commonly used. This has resulted in a parsimonious rather than piecemeal account that predicts additional measures as well. We now briefly explore some other potential applications of the model as well as its limitations.

#### *Limitations and Extensions of the SVM Model*

The SVM model as introduced here is restricted to measures that are based on binary comparison processes. Rating scale measures cannot be derived from such comparisons and are therefore not covered by the present model. Ratings also cannot be derived from abstract preference relations described by standard utility theories of preference (Luce, 2000; Raiffa, 1968). Such measures include attraction ratings for gambles (Goldstein & Einhorn, 1987; Mellers, Chang, et al., 1992; Mellers, Ordoñez, & Birnbaum, 1992), risk ratings (e.g., Mellers & Chang, 1994; Weber, Anderson, & Birnbaum, 1992), and preference strength ratings (Fischer & Hawkins, 1993). Also, other types of reversals, such as those between joint and separate evaluation of options (Hsee, Lowenstein, Blount, & Bazerman, 1999), are beyond the scope of this review, because these reversals are not contingent on the elicitation method.

The SVM model can be extended to explain attenuation of preference reversals with practice (Cox & Grether, 1996; Lindman, 1971). This interesting finding poses a serious problem for theories that assume changes in utilities across measures. If utili-

ties were actually changed across measures, repeated experience should reinforce rather than attenuate the preference reversals, contrary to what is found. The SVM model can explain the observed attenuation with practice by assuming that the response distribution from the preceding practice trial is used as the initial distribution for the ensuing practice trial. Busemeyer and Goldstein (1992) showed how this assumption can account for attenuation of reversals across sessions in a special case of the SVM model, and Jessup et al. (2004) showed how this assumption produces correct SVM model predictions for convergence of pricing measures over repeated trials. These are additional predictions that have not been treated by any of the approaches mentioned earlier; at best, those theories would require yet further changes in the utility mapping—within a particular response method.

Assumptions of the SVM model, as presented herein, can also be reconsidered when appropriate. First, experimental manipulations may justify reasoned modification of model parameters. For example, the current application assumed *judged* CEs that are directly reported. An alternative is *choice-based* CEs, which are elicited by a sequential choice procedure that iteratively narrows down the range of values (e.g., Bostic et al., 1990). The SVM model can easily accommodate this procedure by changing the assumptions about initial values and transitions among candidates to align with those imposed by the experimenter. Second, more general utility mappings are possible in the SVM model through the use of an existing rank dependent or configural weight utility theory to determine the weights in Equation 6. Furthermore, process-tracing techniques (e.g., looking time) may allow us to better estimate the attention weights in the choice model and SVM comparison layer, providing a data-based estimate of weights that could then be used to predict response distributions and times.

#### General Discussion

A new model, called the SVM model, has been introduced for predicting human preferences across a wide variety of measurement methods. A key property that makes this model different from previous explanations of preference reversals is that it focuses on the response mapping rather than the utility mapping. This model is capable of generating predictions for no fewer than six distinct types of responses: choices, buying prices, selling prices, CEs, PEs, and matched payoffs. The SVM model was conceptually designed to accomplish two main classes of tasks. The first task is paired choice, in which accumulation of affective reactions to stimulus properties drives a preference state over time, resulting in choice probabilities for each option. The second task is a sequential search for a value (CE, PE, matched payoff, or matched probability) that elicits indifference between two options.

The SVM model was applied to a particular set of robust inconsistencies in preferential tasks—preference reversals among the different response methods. The key results predicted by the model, in accord with published empirical trends, are (a) classic reversals between choice and pricing measures; (b) the impact on these reversals of unattractive gambles featuring losses, riskless “unidimensional” options such as single-outcome investments with delayed receipt, gambles with equal outcome ranges, and gambles with equally likely outcomes; (c) reversals between different pricing measures, WTP and WTA; (d) reversals between CE and PE methods; and (e) the relation between stimulus variance and response variance. The ability of the SVM model to account

for all of these results is more impressive when we consider the fact that a single model specification and one set of parameters were used for all these applications as well as for successfully predicting multiple (three or four) measures simultaneously. Furthermore, the SVM model was shown to provide an excellent quantitative fit to a particular data set consisting of 168 gambles, reproducing detailed trends in the data, such as patterns of violations of branch independence. The previous section details the inability of other approaches to account for all of these results.

The SVM model also possesses other advantages over competing explanations for preference reversals. First, we have completely specified the SVM model, which means it can be applied to all of the standard preference elicitation methods. Second, the SVM model can account for variability in human behavior that other models cannot. That is, the SVM model makes predictions for novel measures such as entire distributions of reported prices and response times. Third, the SVM model benefits from psychological parsimony by retaining consistent utility mappings. We rely on a single mechanism, which operates the same across all response methods, without changing weights, values, or free parameters. Fourth, this mechanism also formally describes the deliberation and response process, which coincides with empirical process-tracing results. Finally, our approach has been shown to successfully account for many other decision-making phenomena (Busemeyer & Townsend, 1993; Diederich, 2003; Johnson & Busemeyer, 2005; Roe, Busemeyer, & Townsend, 2001). For example, the choice model on which the SVM model is based, DFT, has been shown to account for other types of preference reversals—those between binary and ternary choice sets (Roe et al., 2001) and those induced by time pressure (Diederich, 2003).

#### *Model Details, Parameters, and Tractability*

The evaluation of our model stresses the importance of variance in the stimuli (e.g., gambles) of preferential tasks. With respect to the preference reversal phenomena, this focus has never been fully realized. In fact, Slovic and Lichtenstein (1968) suggested that the variance of gambles plays a minor role, contrary to our claim. The majority of work on preference reversals has focused on the dimensional differences between stimuli—that one gamble excels on the probability dimension, whereas the other has advantages on the payoff dimension. However, Birnbaum and colleagues (e.g., Birnbaum & Sutton, 1992) have stressed the importance of the payoff range, which is directly related to the variance for the equal-probability gambles they used, and Ganzach (1996) also noted the importance of variance in the five-outcome gambles he used. It is undeniable that variance exists in human behavior; we are not always deterministic creatures of habit. The current model realizes this fact by allowing for human psychology (the deliberation process) to be affected by task features (stimulus variance), which in turn produces variance in responses.

The SVM model may appear to be quite complex to those who are more familiar with the typical algebraic utility equations that describe popular theories. In practice, however, the SVM model is simple with respect to parameters. Only three free parameters were needed for all the applications: the discriminability index  $d$ , computed directly from the stimuli<sup>16</sup>; the threshold  $\theta$ , which represents the amount of information required to make a choice; and the exit rate  $r$ , which controls the likelihood of making an indifference response. The most flexible application would entail fitting these

parameters to one task, then holding these parameters constant across all subsequent tasks (although our qualitative applications here instead preset all parameter values). Furthermore, one need not abandon computational models such as the SVM model because of a lack of tractability or precision. Often, economists dismiss psychological models because of their lack of specification. Although Sterman (1987) provided a protocol for empirical testing of dynamic simulation models in economics, simulations are not necessary for deriving SVM model predictions. Rather, precise quantitative predictions for the SVM model are made possible by mathematical theorems computed via matrix methods (Diederich & Busemeyer, 2003). These calculations are easily performed by many available software packages.

#### *Theoretical Issues*

It is important to note that we do not propose sweeping rejection of other mechanisms, such as contingent weighting and valuation. Rather, we see the present model as complementary to other theories. In particular, we have shown that one need not necessarily assume changes in the utility structure. Note that our approach shares with other theories the psychological importance of the decision maker's viewpoint, although it is formulated here in terms of differential starting values (response) rather than differential weighting (evaluation). Basically, we propose that (as theorists) we can let the process do most of the work in explaining human decision-making behavior, as opposed to jumping to the conclusion that weights or values are constructed "on the fly" anew for every change in task and context (see Plott, 1996, for a related argument). Although effects such as compatibility and prominence may indeed influence decision-making behavior—and could be additionally incorporated in the present model, in defining  $d$ —such effects may not always be required to explain robust inconsistencies.

Undoubtedly, situations exist in which inputs such as decision weights do in fact change across contexts. For example, preference reversals between acceptance and rejection decisions (Shafir, 1993) cannot be explained by the current theory without changes in decision weights. Furthermore, changes in decision weights are necessary to explain some patterns of dominance violations, such as those reported in Birnbaum (1992). We simply suggest that a large proportion of empirical phenomena can be accounted for through the exploration of alternative explanations, such as our focus on the response process, and that the influence of factors such as contingent weighting may be more modest than previously assumed.

In conclusion, we stress the importance of using microlevel processes to understand macrolevel behavior. Beyond the successful prediction of particular phenomena with the SVM model, we have illustrated the usefulness of the more general computational modeling approach (see also Busemeyer & Johnson, 2004). Specifically, we have used a dynamic and stochastic computational model, based on principles of sequential information sampling. In the current applications, a computational model of the response process permits us to retain the idea of a coherent value system,

<sup>16</sup> We computed this index using one fixed parameter,  $\alpha$ , in the qualitative application, and two free weighting parameters in the quantitative application.

which had been all but abandoned in light of the empirical results covered herein. Although models of this type may appear more complex (compared with deterministic algebraic equations), the explanatory power to be gained seems well worth the added complexity. Rather than inferring cognitive functioning from overt behavior, we actually model the underlying cognitive mechanisms, then examine the predictions made about behavior. This allows us to formulate a single, comprehensive model for particular domains and phenomena without having to relax assumptions or add “biases” for each new empirical result. We believe this path will lead us to more complete, parsimonious accounts of human decision-making behavior and cognition in general.

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(Appendixes follow)

Appendix A

Mathematical Derivations of Computational Models

This appendix describes the derivations of the predictions reported in the text. For additional details, the reader is referred to Busemeyer and Townsend (1992), Diederich and Busemeyer (2003), and Johnson (2004). Three models are described here: the DFT binary choice model, the SVM comparison model for indifference judgment, and the SVM matching model.

General Formulation

All three models are dynamic systems that can be wholly analyzed in terms of initial states and transition probabilities. First, we define a column vector  $\mathbf{Z}$ , where each element represents the probability  $z_i$  that a dynamic system begins in state  $i$  ( $\sum z_i = 1$ ). Second, we define a square matrix  $\mathbf{T}$ , containing the probabilities  $t_{m,n}$  that the system transits from state  $m$  to state  $n$ :

$$\mathbf{T} = \begin{bmatrix} 0 & t_{1,2} & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ t_{2,1} & 0 & t_{2,3} & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & t_{3,2} & 0 & t_{3,4} & \dots & 0 & 0 & 0 & 0 \\ \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & t_{(k-2),(k-3)} & 0 & t_{(k-2),(k-1)} & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & t_{(k-1),(k-2)} & 0 & t_{(k-1),k} \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & t_{k,(k-1)} & 0 \end{bmatrix}$$

Third, we define a diagonal absorption matrix  $\mathbf{A}$ , containing the probabilities  $a_f$  that the dynamic system transits into the final state  $f$ . Finally, the column vector  $\mathbf{R}$  is used to denote the final probability of selecting each of the possible responses, which is computed from the matrix equation:

$$\mathbf{R} = \mathbf{Z}'[\mathbf{I} - \mathbf{T}]^{-1}\mathbf{A}. \tag{A1}$$

Dynamic systems also naturally produce response time predictions. When we specify a time unit  $h$ , the mean response times required to make each response are similarly contained in the mean response time vector  $\mathbf{S}$ ,

$$\mathbf{S} = h(\mathbf{Z}'[\mathbf{I} - \mathbf{T}]^{-2}\mathbf{A})/\mathbf{R}, \tag{A2}$$

where  $\mathbf{X}/\mathbf{Y}$  denotes element-wise division of two matrices with the same dimensionality.

DFT Binary Choice Model

This model is used for a binary choice task between two gambles, F and G (refer to Figure 2a). All transition probabilities in  $\mathbf{T}$  and  $\mathbf{A}$  are defined in terms of  $p$  or  $q$ , as computed via Equation 4 in the text. Binary choice produces only two nonzero final absorption states: the lower absorbing State 1 (choose G) with transition probability  $a_1 = q$  in the first diagonal element of  $\mathbf{A}$ , and the upper absorbing State  $k$  (choose F) with transition probability  $a_k = p$  in the last diagonal element of  $\mathbf{A}$ . For the remaining intermediate diagonal elements of  $\mathbf{A}$ ,  $a_f = 0$  for  $f \neq 1, k$ . The transitions between intermediate states in  $\mathbf{T}$  are  $t_{1,2} = p$  and  $t_{k,(k-1)} = q$  for the first and last rows of  $\mathbf{T}$ , respectively, and  $t_{m,(m+1)} = p$  and  $t_{m,(m-1)} = q$  for all other rows  $m \neq 1, k$ . The choice process is unbiased and begins in the neutral state. If we use an odd value of  $k$ , this is the state  $s = (k+1)/2$ , so that  $z_s = 1$ . With this specification, the final choice probabilities are given by Equation A1, with the probability of choosing G in the first row,  $R_1$ , and the probability of choosing F in the last row,  $R_k$ , respectively, of the response vector  $\mathbf{R}$ .

SVM Comparison Layer

This model is used for a choice between a gamble—for instance, F—and a candidate value—for instance,  $C_r$ —but now with three possible responses (see Figure 3b): choose the gamble (causing an increment in the candidate value), choose the candidate value (causing a decrement in the candidate value), or respond “indifferent” (stop and report the candidate value). This process begins unbiased, as in the choice model,  $z_s = 1$ . The transition probabilities in  $\mathbf{A}$  and  $\mathbf{T}$  for the SVM comparison layer now depend on the candidate value,  $C_i$ , that is being evaluated. In particular, the discriminability index used to compute transition probabilities  $p$  and  $q$  via Equation 4 are now determined by replacement of Gamble G with the sure value  $C_i$  (with Equations 1b and 2b substituted for Equations 1 and 2). The transition matrices  $\mathbf{A}$  and  $\mathbf{T}$  are defined in the same way as for the DFT binary choice model, with the following exceptions for the indifference response. The indifference response occurs with probability  $r$  whenever the system enters the neutral state. Therefore, the absorption matrix  $\mathbf{A}$  is modified to include  $a_s = r$  (allowing transitions from the neutral state) and the transition matrix  $\mathbf{T}$  is modified accordingly as  $t_{s,s+1} = (1 - r)p$  and  $t_{s,s-1} = (1 - r)q$ . Using these modifications of the transition probabilities in  $\mathbf{A}$  and  $\mathbf{T}$ , we can then compute the probabilities of the three responses using Equation A1. The probability of choosing the candidate  $C_j$  will then appear in the first row 1 of  $\mathbf{R}$ , the probability of choosing the gamble will appear in the last row  $k$  of  $\mathbf{R}$ , and the probability of an indifference response appears in the middle row  $s$  of  $\mathbf{R}$ . We explicitly denote the dependence of these three choice probabilities on the candidate value  $C_i$  and introduce shorthand notation as follows:

$$\begin{aligned} \text{Pr[choose candidate } C_i \text{ over Gamble F]} &= R_1 = \delta^+(C_i), \\ \text{Pr[choose Gamble F over candidate } C_i] &= R_k = \delta^-(C_i), \text{ and} \tag{A3} \\ \text{Pr[indifference between Gamble F and candidate value } C_i] &= R_s = \delta^0(C_i). \end{aligned}$$

SVM Matching Layer

This model (Figure 3a) is used to select a candidate value from a set of  $n$  candidates,  $\mathbf{C} = \{C_1, \dots, C_n\}$ . All of the transition probabilities in  $\mathbf{A}$  and  $\mathbf{T}$  are now determined by the choice probabilities,  $\delta^+(C_i)$ ,  $\delta^-(C_i)$ , and  $\delta^0(C_i)$  defined earlier in Equation A3. The transition probabilities in  $\mathbf{T}$  are as follows:  $t_{1,2} = 1 - \delta^0(C_1)$  and  $t_{k,(k-1)} = 1 - \delta^0(C_n)$  for the first and last candidate values, and  $t_{m,(m+1)} = \delta^+(C_i)$ , and  $t_{m,(m-1)} = \delta^-(C_i)$  for all candidates  $C_{i \neq 1,n}$  (i.e., for rows  $m \neq 1, k$ ). The responses of the matching layer are the  $n$  candidate values in  $\mathbf{C}$ . The transition probabilities in the absorption matrix,  $\mathbf{A}$ , are the probabilities of indifference computed in the comparison layer for each of these values. That is, the probability of transiting to the final state,  $f$ , or reporting candidate  $C_f$  is defined by  $\delta^0(C_f)$  in the comparison layer:  $a_f = \delta^0(C_f)$  for all candidates  $C_f$ .

The initial state vector  $\mathbf{Z}$  of the SVM matching layer defines the initial probability distribution over the set of  $n$  candidate values. This initial distribution is described in Footnote 5 and in the next section, where we discuss parameters (see also Figure 4). As before, we compute the response probability distribution for the final candidate prices from Equation A1. For the SVM matching layer, row  $m$  of  $\mathbf{R}$  indicates the final probability with which candidate  $C_m$  is reported as the matched value.

Parameters

Our models are conceptually continuous dynamic systems that have been approximated by a discrete representation for ease of computation. As

a result, we prefer to think of the parameters in three classes. The first are control parameters exclusively used to give the discrete approximations—these are the time unit  $h$  and the dimensionality  $k$ . We set the time unit to a small value,  $h = 0.10$ , across all applications. This, in turn, determines the discrete step size, or distance between adjacent preference states in the choice model ( $\Delta^2 = h$ ) for use in Equation 4. We use  $k = 19$  preference states in the choice model and  $n = 21$  candidate values in the SVM model for all applications. Sensitivity analyses from Diederich and Busemeyer (2003) confirm that these parameters give the desired degree of precision (e.g., choice probabilities precise to two decimal places).

The second class includes those parameters preset via assumptions of the model. The initial states of the DFT choice model and SVM comparison layer are unbiased, as mentioned above. The initial candidate distribution,  $\mathbf{Z}$  in the SVM matching layer, is modeled via a binomial distribution with  $k = 21$  values and a mode of 0.1, 0.9, or 0.5 for WTP, WTA, and all other methods, respectively. We assume the candidate values  $C_1$  and  $C_k$  are

determined by the input gamble. For pricing, these are the minimum and maximum gamble outcomes, respectively; for payoff matching, these are zero and the maximum of the comparison gamble, respectively. For the PE, we simply assume the range  $[0, 1]$  and compute candidate values via the expected utility equations in the text.

Practically, there remain (at most) three free parameters for the SVM model that could be fit to empirical data: the exit rate  $r$ , the threshold  $\theta$ , and the discriminability index  $d$ . Here, the exit rate was held constant across qualitative applications,  $r = .02$ , and was fit as a free parameter in the quantitative application. The threshold of the DFT choice model (and SVM comparison layer) is redundant, because of the control parameters we chose:  $\theta = \Delta(k - 1)/2 \approx 3$ . The discriminability index is also determined by the input gambles, via the equations provided in the text.

Further details about parameter values and rationale for parameter selection can be found in the text. Detailed procedures used for deriving predictions, as well as the associated MATLAB routines, are available on request from us.

### Appendix B

#### Contingent Valuation Application to Pricing (WTP vs. WTA) Reversals

This appendix assesses the ability of contingent valuation, as described in the text, to account for reversals between buying and selling prices. Birnbaum and Zimmermann (1998) have also shown the failure of various contingent valuation models to account for these reversals. In this section, we assume gambles with two equiprobable outcomes,  $x$  and  $y$ , that have been shown to produce within-pricing reversals (Birnbaum & Sutton, 1992). This makes the utility for a gamble, without probability weighting,<sup>B1</sup> equal to

$$[u(x)/2 + u(y)/2]. \tag{B1}$$

We first assume monotonically increasing utility,  $u$ , and a simple form of loss aversion where losses are perceived to be some magnitude greater than equal gains (Tversky & Kahneman, 1991):

$$u(-x) = -k \times u(x). \tag{B2}$$

When the decision maker is asked for a buying price, the reference point suggests evaluating the price paid,  $B$ , as a loss and the possible gamble outcomes,  $x$  and  $y$ , as gains. The task involves equating this loss and gain by reporting a value,  $B$ , that satisfies

$$[u(x)/2 + u(y)/2] + u(-B) = 0, \tag{B3}$$

which implies, through loss aversion in Equation B2,

$$[u(x)/2 + u(y)/2] = k \times u(B), \tag{B4}$$

or

$$u(B) = (1/k)[u(x)/2 + u(y)/2]. \tag{B5}$$

When the decision maker is asked for a selling price, the reference point suggests evaluating the price received as a gain and the foregone gamble outcomes as losses. The task involves reporting a selling price,  $S$ , producing similar indifference between the price and gamble:

$$u(S) + [u(-x)/2 + u(-y)/2] = 0, \tag{B6}$$

which results again, through loss aversion, in

$$u(S) = k \times [u(x)/2 + u(y)/2]. \tag{B7}$$

Thus, we obtain by combining Equations B5 and B7

$$u(S) = k^2 \times u(B). \tag{B8}$$

Finally, note that because  $u$  is assumed to be monotonically increasing,  $u^{-1}$  is therefore monotonically increasing. By taking this inverse over Equation B8,

$$S = u^{-1}[k^2 \times u(B)], \tag{B9}$$

we see that the selling price is monotonically related to the buying price, which cannot produce reversals in rank orders across these two methods. Thus, shifting reference points and loss aversion cannot explain reported reversals between WTP and WTA.

<sup>B1</sup>Allowing any probability weighting does not change the basic result.

Received July 28, 2004  
 Revision received May 20, 2005  
 Accepted May 24, 2005 ■