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Running head: QUANTUM INFORMATION PROCESSING THEORY

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Quantum Information Processing Theory

Synonyms

Applied quantum probability theory

Definition

Quantum information processing theory is an alternative mathematical approach for generating theories of how an observer processes information. Typically, quantum information processing models are derived from the axiomatic principles of quantum probability theory. This probability theory may be viewed as a generalization of classic probability. Quantum information processing models do not make assumptions about the biological substrates. Instead this approach provides new conceptual tools for constructing social and behavioral science theories.

Theoretical Background

There are two mathematical approaches to constructing probabilistic system: classic Kolmogorov probabilities and quantum von Neumann probabilities. The majority of information processing models in cognitive science and psychology use the classical probability system. However, classic probability and information processing theory impose a restrictive set of assumptions on the representation of complex systems such as the human cognitive system. Quantum information processing theory postulates a more general method for representing and analyzing these types of complex systems. We begin by providing motivation for adopting the quantum approach and then give a mathematical comparison of classical and quantum probability theories.

Reasons to adopt the quantum probability framework for cognitive science

(1) Cognitive measures create cognitive states rather than record states. For example, suppose you are trying to understand the emotional state of a person after the presentation of an arousing stimulus. Classical information processing theory posits that the cognitive system of the individual is in a definite state before any measurements are taken. The process of imposing the measurement has no effect on this state other than to simply record it. On the other hand, quantum information processing theory postulates that the state of the cognitive system is undetermined before measurement, and it is the process of imposing measurements that determines the state. Thus, judgment is not a simple read out from a pre-existing or recorded state, instead it is constructed from the current context and question.

(2) Cognition behaves like a wave rather than a particle. For example, suppose you are a juror deciding whether a defendant is guilty or not. Classical information processing theory assumes your cognitive system is always in one of two states, guilty or innocent. So, at any given moment in time, your cognitive state is clearly known to you. However, quantum theory suggests that beliefs are superimposed and you do not jump from one state to another. Thus, you can feel a sense of ambiguity about all of the states simultaneously.

(3) Cognitive measures disturb each other, creating uncertainty. For example, suppose I question you about your preferences for cars. In one scenario, I ask you directly the type of car you would like to buy, and you respond with a specific preference. However, in another scenario, I first ask what type of car your spouse would like to buy. Then, when I ask you about your own preferences, they become less certain. Classical theories cannot capture the effects of measurement disturbances. However, using quantum information processing theory, questions can be represented as incompatible thus allowing for one question to disturb the answer to another.

(4) Cognitive logic does not obey classic logic. Returning to the jury decision-making example, you might believe the defendant is guilty or innocent, and you might also feel that the defendant is good or bad. According to classic logic, the distributive axiom yields:

$$Guilty \wedge (Good \vee Bad) = (Guilty \wedge Good) \vee (Guilty \wedge Bad)$$

On the other hand, quantum theory allows for the existence of a superimposed state, and does not always obey the distributive axiom:

$$Guilty \wedge (Good \vee Bad) \neq (Guilty \wedge Good) \vee (Guilty \wedge Bad)$$

Thus, quantum logic is more generalized than classical logic and can model human judgments that do not obey Boolean logic.

If we think of the four points above in terms of 'physical measurements' and 'physical systems' instead of 'human judgments' and 'cognitive systems', then these points are similar to the ones that faced physicists in the 1920's that forced them to develop quantum theory. In other words, physical findings that seemed paradoxical from the classical point of view led physicists to invent quantum theory. Similarly, paradoxical findings in cognitive psychology suggest that classical probability theory is too limited to fully explain various aspects of human cognition. These phenomena include violations of the sure thing axiom of decision making, violations of the conjunctive and disjunctive rules of classic probability theory, and cooperation in Prisoner Dilemma games.

A formal comparison of classical and quantum probability theories

Classical probability theory postulates a set of all possible outcomes Ω called the outcome space. For example, when flipping a coin the outcome space $\Omega = \{head, tail\}$.

Events are defined as subsets of the Ω and correspond to things that might or might not happen. New events can be formed from other events in three different ways. Specifically, let x, y, and z be events in the outcome space Ω. The negation operator, $\neg x$, denotes the complement of x. The conjunction operator, $x \wedge y$, denotes the the intersection of x and y. The disjunction operator, $x \vee y$, denotes the union of x and y. Since events are mathematically represented as sets, they obey the rules of Boolean algebra:

1. Commutative: $x \vee y = y \vee x$

2. Associative: $x \vee (y \vee z) = (x \vee y) \vee z$

3. Complementation: $x \vee (\neg y \wedge y) = x$

4. Absorption: $x \vee (x \wedge y) = x$

5. Distributive: $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$

A probability distribution, Pr, is a function of events that obeys the Kolmogorov axioms:

1. Non-negative: $Pr(x) \geq 0$

2. Addition: If $x_1, ..., x_n$ is a partition of x, then $Pr(x) = Pr(x_1) + ... + Pr(x_n)$

3. Total one: $Pr(\Omega) = 1$

Quantum probability theory replaces the outcome space Ω with a Hilbert space H (i.e. a complex vector space). Quantum events are defined geometrically as a subspace (e.g. a line or place, etc.) within this Hilbert space. Similar to classical probability theory, new events can be formed in three ways. Let L_x, L_y and L_z represent three different events in H. The negation operator, $L_x^\perp$, denotes the maximal subspace that is orthogonal to L_x. The meet operator, $L_x \wedge L_y$, denotes the intersection of the two subspaces, L_x and L_y. The join operator, $L_x \vee L_y$, denotes the span of L_x and L_y. Quantum logic obeys all of the rules of Boolean logic except for the distributive axiom:

$$L_x \wedge (L_y \vee L_z) = (L_x \wedge L_y) \vee (L_x \wedge L_z).$$

Quantum probability postulates the existence of a unit length state vector $|z\rangle \in H$. (We use Dirac, or Bra-ket, notation in keeping with the standard notation used in quantum mechanics. For our purposes, $|x\rangle$ denotes a column vector whereas $\langle x|$ denotes a row vector.) This state vector depends on the context of the situation being modeled. For example, the state vector might represent an individual's cognitive state during a decision-making task. Quantum probabilities are computed by projecting $|z\rangle$ onto subspaces representing events. Specifically, for each event L_x there is a corresponding projection operator P_x that projects $|z\rangle$ onto L_x. The probability of the event L_x is equal to the squared length of this projection:

$$Pr(L_x) = |(P_x|z\rangle)|^2$$

Since events are mathematically represented as subspaces, there exist a basis for each event. For example, let $|x_1\rangle, ..., |x_n\rangle$ be a set of basis vectors for the event corresponding to the subspace L_x. This is similar to a partition of an event in classical probability theory. When selecting a basis for L_x, we choose one that is orthonormal: inner products $\langle x_i|x_j\rangle = 0$ and lengths $|\langle x_i|x_j\rangle| = 1$. Now, we can use this basis to constructor projectors. The projector, P_{x_i}, projects the state vector $|z\rangle$ onto the subspace L_{x_i}, and it is constructed from the outer product $P_{x_i} = |x_i\rangle\langle x_i|$. The projector, P_x, for the event L_x is constructed by adding the projects corresponding to the basis vectors:

$$P_x = P_{x_1 \vee ... \vee x_n} = P_{x_1} + ... + P_{x_n} = |x_1\rangle\langle x_1| + ... + |x_n\rangle\langle x_n|.$$

This allows us to write the probability of the L_x as a sum of separate probabilities:

$$Pr(L_x) = |(P_x|z\rangle)|^2 = |(|x_1\rangle\langle x_1| + ... + |x_n\rangle\langle x_n|)|z\rangle|^2$$

$$= ||x_1\rangle\langle x_1|z\rangle + ... + |x_n\rangle\langle x_n|z\rangle|^2 = |\langle x_1|z\rangle|^2 + ... + |\langle x_n|z\rangle|^2$$

where the final step follows from the orthogonality property. Finally for any state vector $|z\rangle$ we have that $P_H \cdot |z\rangle = |z\rangle$ showing $|P_H \cdot |z\rangle|^2 = |\langle z|z\rangle|^2 = 1$. From this we see that $P_H = \sum_i^m P_{x_i} = \sum_i^m |x_i\rangle\langle x_i| = I$ where $|x_1\rangle, ..., |x_m\rangle$ is a basis for the Hilbert space H and I is the identity operator.

From these properties we see that quantum probabilities obey rules analogous to the Kolmogorov rules:

1. Non-negative: $Pr(L_x) = |(P_x|z\rangle)|^2 \geq 0$

2. Addition: If $x_1, ..., x_n$ is a basis of L_x and $L_{x_i} \wedge L_{x_j} = 0$ for all i, j, then

$$Pr(L_x) = Pr(L_{x_1}) + ... + Pr(L_{x_n})$$

3. Total one: $Pr(H) = 1$

Another important concept in quantum probability theory is the notion of superposition and quantum measurement. Any state vector $|z\rangle$ prior to measurement can be expressed in terms of the basis states as follows:

$$|z\rangle = I \cdot |z\rangle = \sum_i^m |x_i\rangle\langle x_i| \cdot |z\rangle = \sum_i^m |x_i\rangle\langle x_i|z\rangle = \sum_i^m \langle x_i|z\rangle \cdot |x_i\rangle.$$

Since the inner product called the probability amplitude $\langle x_i|z\rangle$ is a scalar, we see that state vector is a linear combination, or superposition, of the basis states. The act of measurement results in the normalized projection of the state vector onto the corresponding subspace. For example, if we perform measurement X, this measurement changes the initial state $|z\rangle$ to a new state $|(z|x))\rangle$ achieved by projecting $|z\rangle$ onto the subspace corresponding to X, L_x. Then the projection is normalized so the new state has unit length.

When taking more than one measurement of a state vector, quantum theory allows for the measurements to either be compatible or incompatible. Intuitively, compatibility

means that measurements X and Y can be accessed simultaneously or sequentially without interfering with each other. On the other hand, if X and Y are incompatible, they cannot be accessed simultaneously. From a cognitive standpoint, this implies that the two measurements are processed serially and one measurement interferes with the other. Mathematically, incompatible measurements, or events, are represented by different bases for the same n-dimensional subspace. For example, if measurement X is represented by the subspace L_x with basis $|x_1\rangle, ..., |x_n\rangle$ and measurement Y is represented by the subspace L_y with basis $|y_1\rangle, ..., |y_n\rangle$, the $|x_i\rangle$ basis is a linear transformation of the $|y_i\rangle$ basis. If X and Y are compatible, then there is one basis representation for both measurements. In this case, quantum probability theory reduces to classic probability theory (Hughes, 1989).

Although classical and quantum probability theory share many features such as Kolmogorov-like rules, these two theories have important differences. First, quantum theory does not necessarily obey the distributive axiom of Boolean logic. Also, quantum probability postulates that the state of a system is a superposition of all possible states before measurement, and it is the act of measurement that changes the state. Finally, quantum theory allows for both compatible and incompatible measurements whereas classical probability theory assumes all measurements are compatible.

Important Scientific Research and Open Questions

To illustrate how quantum information processing theory can be applied to empirical data, we draw upon a categorization and decision-making study conducted by Busemeyer, Wang, et Lambert-Mogiliansky (2009). In this experiment, participants were shown pictures of faces that varied on two dimensions (face shape and lip thickness) and were asked to categorize the faces as good guys or bad guys and to decide to act friendly or aggressive. In one condition, participants made an action decision without reporting a category. In a second condition, participants made an action decision after categorizing

the face. In the first condition, we can think of the categorization task as a hidden

inference. Thus, by the law of total probability from classical probability theory,

$$Pr(aggressive \mid face) = Pr(good \mid face) \cdot Pr(aggressive \mid good) + Pr(bad \mid face) \cdot Pr(aggressive \mid bad).$$

However, data collected from the experiment does not conform to the law of total

probability. From the first condition, Busemeyer et al. found

$Pr(aggressive \mid face) = 0.69$. Then, from the second condition, Busemeyer et al. found

$Pr(good \mid face) = 0.17$, $Pr(aggressive \mid good) = 0.42$, $Pr(bad \mid face) = 0.83$, and

$Pr(aggressive \mid bad) = 0.63$. By applying the law of total probability to the data from the

second condition, we have

$$Pr(aggressive \mid face) = 0.17 \cdot 0.42 + 0.83 \cdot 0.63 = 0.59.$$

Obviously, this does not match the data collected in condition 1.

Now, we provide a quantum probability explanation for the data. Let $\langle good|face \rangle$

be the amplitude for transiting from the face stimulus to a categorization of good. Thus,

$|\langle good|face \rangle|^2$ is the probability of a face to 'good guy' transition. Since the only two

categorization responses are good and bad, we also have $|\langle good|face \rangle|^2 + |\langle bad|face \rangle|^2 = 1$.

Similarly, let $\langle aggressive|good \rangle$ be the amplitude for transiting from the 'good guy'

categorization to the aggressive action decision. Then, the probability of this transition is

$|\langle aggressive|good \rangle|^2$ and $|\langle aggressive|good \rangle|^2 + |\langle friendly|good \rangle|^2 = 1$. Thus, we have

$$Pr(aggressive \mid face) = |\langle aggressive|face \rangle|^2$$

$$= |\langle good|face \rangle \langle aggressive|good \rangle + \langle bad|face \rangle \langle aggressive|bad \rangle|^2$$

$$= |\langle good|face \rangle \langle aggressive|good \rangle|^2 + |\langle bad|face \rangle \langle aggressive|bad \rangle|^2$$

$$+2 \cdot |\langle good|face \rangle \langle aggressive|good \rangle \langle bad|face \rangle \langle aggressive|bad \rangle| \cdot cos(\theta)$$

where $cos(\theta)$ is the <u>interference effect</u> term. From this we see that quantum theory does not obey the law of total probability. So, we can find a value for θ such that the quantum probability model matches the experimental data.

Researchers are currently working towards building unified theories of human cognition based on the principles of quantum theory. To date, quantum information processing models have been used to explain cognitive phenomena including violations of rational decision making principles, paradoxes of conceptual combination, human judgments, perception, and order effects on human inference.

Cross-References

- Human information processing
- Mathematical models/theories of learning
- Modeling and simulation
- Quantum analogical modeling

Références

Busemeyer, J. R., Wang, Z., & Lambert-Mogiliansky, A. (2009). Empirical comparisons of markov and quantum models of decision making. *Journal of Mathematical Psychology*, *53*, 423–433.

Hughes, R. I. G. (1989). *The structure and interpretation of quantum mechanics*. Harvard University Press.

Definitions

- interference effect - Empirical violations of the law of total probability are termed interference effects. These effects can be found both in physical systems and human cognitive systems. Quantum theory was initially invented to explain these effects in particle physics. Psychologist have also found interference effects in humans, motivating researchers to apply quantum theory to cognitive science.