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Modeling Dynamic Inconsistency with a Changing Reference Point

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ABSTRACT

A rational principle of decision making called dynamic consistency was tested by presenting decision makers with a sequence of two gambles. The first gamble was obligatory. Before playing the first gamble, participants were asked to make a planned choice as to whether they would take the second gamble. After experiencing the actual results of the first gamble, decision makers were asked to make a final choice regarding the second gamble. Dynamic consistency requires agreement between the planned and final choices. Violations of dynamic consistency were observed, e.g. anticipating a gain in the first gamble, decision makers planned to take the second gamble; after experiencing the gain, they changed their minds and rejected the second gamble. Two models of dynamic inconsistency were compared. One assumes that experience shifts the reference point and changes the utility associated with the gamble; another assumes that experience changes the subjective probability associated with the gamble. The reference point model provided the best account for the findings. Copyright © 2003 John Wiley & Sons, Ltd.

KEY WORDS dynamic consistency; preference reversal; reference point; isolation-integration

Most real-life decisions require decision makers to plan for an uncertain future before they decide how to act in the present. For example, a student must plan for a future job when deciding what courses to take now while she is in college. As another example, a senior manager must plan ahead for future markets when deciding what products the company should begin developing now. The generally prescribed procedure for planning a path for future action entails backward induction (cf. von Winterfeldt & Edwards, 1986; Keeney & Raiffa, 1976; Raiffa, 1968). This procedure requires the decision maker (DM) to attach utilities and probabilities to future outcomes, compute the expected utility of the different paths, and choose the path that maximizes expected utility. One of the central assumptions of backward induction is a principle called dynamic consistency (Sarin & Wakker, 1998; Machina, 1989).

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Dynamic consistency is the bridge that connects decision analysis with actual choices. According to this principle, the preferences upon which a plan is built are stable, and should persist, while carrying out the steps of the plan. Thus, the DM is expected to execute the plan to the letter and to follow the planned path of action. Violation of dynamic consistency would mean that actual experience of anticipated outcomes elicits a change in the utilities of these outcomes and in the related preferences of the DM. If this were the case, then the prescriptive value of backward induction as a planning tool and decision aid would be markedly decreased.

AN EMPIRICAL TEST OF DYNAMIC CONSISTENCY

Barkan and Busemeyer (1999) demonstrated a violation of dynamic consistency utilizing a sequential gambling paradigm (originally developed by Tversky & Shafir, 1992). Barkan and Busemeyer (1999) presented DMs with two-stage decision problems consisting of two sequential gambles (see Figure 1). The first gamble was obligatory, and the DMs could either win or lose points with equal probability. Before playing the first gamble, the DMs had to make a *plan* as to whether or not they would accept a second identical gamble. Planned choices for the second gamble were made contingent on each possible outcome (gain and loss) of the first gamble. After playing the first gamble and actually experiencing the outcome, DMs made a second *final* choice regarding the second gamble.

Barkan and Busemeyer (1999) found that in 20% of the trials, the DMs' final choices were inconsistent with their planned choices. Moreover, these changes of plan were systematic in their direction, and depended on the experienced outcome. One direction of inconsistency showed a tendency towards risk aversion after experiencing a gain. That is, when considering winning the first gamble, a DM made a planned choice to take the second gamble. However, after actually experiencing the anticipated gain, the same DM reversed his or her initial decision and chose not to take the second gamble. Inconsistency also showed a second direction of a tendency towards risk seeking after experiencing loss. That is, when considering losing the first gamble, a DM made a planned choice not to take the second gamble. However, after experiencing the anticipated loss, that same DM reversed his/her decision and made a final choice to take the second gamble.

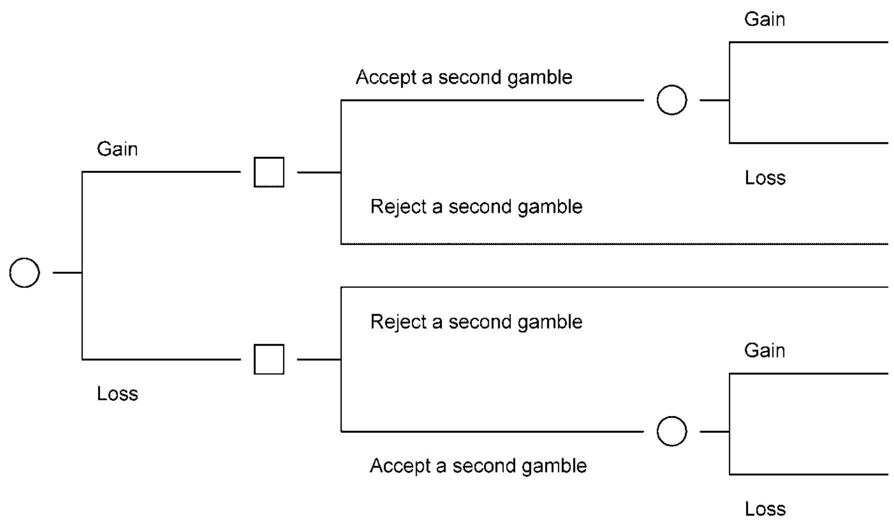

Figure 1. A decision tree representing the sequential gambling paradigm. Each gamble is represented as a chance event (with a circle). Two decision nodes (squares) represent the choice between accepting and rejecting the second gamble

THREE ALTERNATIVE EXPLANATIONS FOR DYNAMIC INCONSISTENCY

Barkan and Busemeyer (1999) considered three alternative explanations for the findings of dynamic inconsistency. One explanation is that dynamic inconsistency simply reflects choice inconsistency—which is random fluctuations in preferences for the same gamble presented twice. However, they proved that according to choice inconsistency, both experienced outcomes (gain or loss) should lead to equal frequencies of risk aversion and risk-seeking inconsistencies. Thus, choice inconsistency (alone) cannot explain the pattern of systematic directions of inconsistency and their dependence on the specific experienced outcome.

A second explanation follows Tversky and Shafir's (1992) analysis of the disjunction effect. This violation of the sure-thing principle was demonstrated in a sequential gambling paradigm. When DMs imagined either a gain of \$200 or a loss of \$100 in a first gamble, they were willing to accept a second identical gamble. However, when they imagined the outcome of the first gamble was unknown, they rejected the second gamble. Tversky and Shafir (1992) suggested that the violation of the sure-thing principle resulted from different evaluations of the second gamble. Imagining either a gain or a loss, evaluations were made by incorporating the imagined outcome from the first gamble. In the face of an unknown outcome, the evaluation was made without incorporating any information from the first gamble. The different evaluations result in different utilities and thus lead to different preferences.

Extending Tversky and Shafir's argument to the dynamic situation at hand suggests that DMs incorporate the outcome of the first gamble in the final evaluation of the second gamble but not in the planned evaluation. We assume that instead of considering the second gamble against the possible outcomes of the first gamble, the DM considers the second gamble against his or her current position. The planned evaluation is made against a current position of zero (since nothing has yet been won or lost). The final evaluation is made against a different position corresponding to an actual gain in the first gamble or to an actual loss in the first gamble.

This explanation suggests that that the reference point used to evaluate the second gamble during the planned stage is different from the reference point used for the evaluation of the second gamble in the final stage. To be specific consider Figure 2. Define X as the monetary value of one of the possible payoffs (win \$200 or lose \$100) of the second gamble; define $u_{\text{plan}}(X)$ as the utility function used to evaluate the payoffs of the second gamble in the planned stage; define $u_{\text{gain}}(X)$ as the utility function used to evaluate the payoffs of the second gamble in the final stage after an actual experience of a gain of \$200; and define $u_{\text{loss}}(X)$ as the utility function used to evaluate the payoffs of the second gamble in the final stage after an actual experience of loss of \$100. The reference point refers to the value of X that is assigned zero utility by a utility function, and this is also the point of inflection on the Prospect Theory utility function.

For the planned choice, we naturally assume the current position is zero and thus, $u_{\text{plan}}(X) = u(X)$. The reference point for this evaluation is also zero because $u_{\text{plan}}(0) = u(0) = 0$. If the first stage yields a gain of \$200, then the 'current' position shifts up. The payoff X from the second gamble is added to the experienced gain and is defined now as $u_{\text{gain}}(X) = u(200 + X)$. Note that the utility function, u_{gain}, is shifted to the left. The reference point for this gain-based evaluation is -200 since $u_{\text{gain}}(-200) = u(200 + (-200)) = u(0) = 0$. The upward shift in the 'current' position results in a shift of the reference point down. If the first stage yields a loss of \$100, then the 'current' position shifts down. The payoff X from the second gamble is added to the experienced loss and is defined now as $u_{\text{loss}}(X) = u(-100 + X)$. The utility function, u_{loss}, is shifted to the right. The reference point for this loss-based evaluation is 100 since $u_{\text{loss}}(100) = u(-100 + 100) = u(0) = 0$. Here the downward shift in the 'current' position results in a shift of the reference point up.

In short, when planning, the DM evaluates the second gamble against a neutral reference point. Experiencing a gain in the first gamble moves the reference point down, so that the payoffs for the second stage gamble lie in the concave or risk averse part of the utility function. When making the final choice, the DM re-evaluates the second gamble against the new reference point. The re-evaluation would cause the same gamble to appear less attractive than it did previously and may lead the DM to reject it. Experiencing a loss in the first gamble moves the reference point up, so that the payoffs of the second stage gamble lie more in the

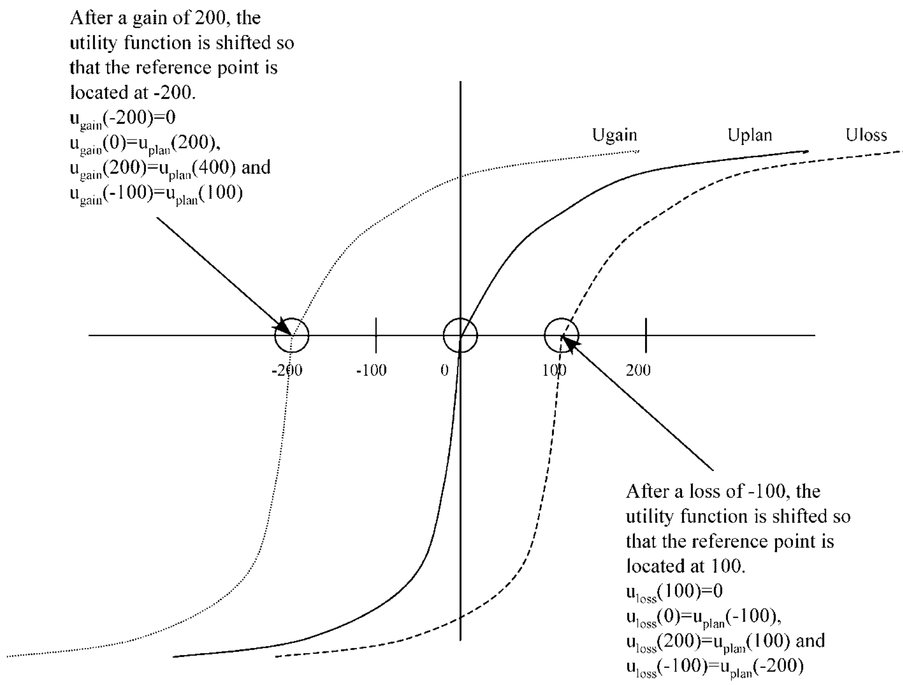

Figure 2. A schematic representation of the change in the reference point. Uplan is the utility function used during the planning stage. The reference point for the planned evaluation is located at the inflection of the function. A gain shifts the current position upwards. The utility function, Ugain, is shifted to the left, and the reference point is shifted downward. A loss shifts the current position down. The utility function Uloss is shifted to the right, and the reference point is shifted upward

convex or risk seeking part of the utility function. Re-evaluating the second gamble against the new reference point would make it seem more attractive than before and could lead the DM to accept it.

The change in the reference point resembles the isolation–integration effect (Kahneman & Tversky, 1979). That is, though decision makers are requested explicitly to consider the possible outcomes of the first gamble and make a plan for possible gain and for possible loss, they are unable to do so. Instead, during planning they evaluate the utility of the second gamble in isolation. After experiencing the first gamble, its outcome is incorporated into the reference point. Thus, only when re-evaluating the utility of the second gamble after experience do the decision makers integrate the prior outcome with the future gain or loss. There are two main differences between the reference-point explanation and the isolation–integration effect. First, the isolation–integration effect refers to static decisions whereas the reference-point explanation refers to a dynamic situation of repeated choices. Second and more importantly, the isolation–integration effect refers to the phrasing of the decision problem (i.e. phrasing the same problem in different ways may lead to different preferences). The reference-point explanation refers to a situation in which the phrasing is constant and the preference reversal is caused by actual experience.

A third explanation is based on changes in subjective probability rather than in the utility associated with the second gamble. According to this explanation, experience triggers a change in the subjective probability in a way resembling the gambler's fallacy. When planning, the DM considers the stated probabilities for winning and losing the second gamble. However, experiencing a gain in the first gamble, would lead to a decrease in the subjective probability associated with another gain (in the second gamble). Re-evaluating the second gamble with decreased subjective probability for winning would make the same gamble appear less attractive than before and could lead the DM to reject it. The opposite would happen after experiencing a loss, since the subjective probability for another loss (in the second gamble) would decrease, and the subjective probability for a gain would increase. Re-evaluation of the second gamble would make it seem more attractive than before and could lead the DM to accept it.[1]

Both the reference-point explanation and the subjective probability explanation can qualitatively produce the systematic directions of dynamic inconsistency observed across subjects. However, a more meaningful comparison would involve modeling the two explanations at the individual level. Such a comparison requires estimates of each DM's utility function and subjective probabilities. Barkan and Busemeyer (1999) argued in favor of the changing reference point model over the changing subjective probability model. However, the design of their initial study did not provide sufficient leverage to rigorously compare these two explanations at the individual level. Their initial demonstration included four basic decision problems and did not permit a reliable assessment of the individual utility functions. Thus, the cause of the systematic directions of inconsistency remains to be determined more convincingly.

EXPERIMENT

The experiment reported below tested the reference-point explanation. This explanation was compared to the two other alternative explanations suggested by Barkan and Busemeyer (1999). As discussed above, the choice-inconsistency explanation cannot qualitatively produce the systematic directions of dynamic inconsistency. However, this explanation serves as a baseline that should be exceeded by the reference-point explanation. To support the reference-point explanation, it should also exceed a viable candidate. The subjective-probability explanation can qualitatively produce the systematic directions of dynamic inconsistency, and allows a more strict comparison. To test the reference-point hypothesis rigorously, we extended the design of the Barkan and Busemeyer (1999) study to include a wider range of decision problems. Furthermore, the present design permits examination of the three alternative explanations at the individual level of analysis. The experiment reported below utilized the sequential gambling paradigm (see Figure 1) with a broad set of 16 decision problems. Each decision problem consisted of two identical gambles. Each gamble gave 50% chances to win or lose points. The first gamble in each decision problem was obligatory. DMs were asked to decide whether or not they would take the second gamble. They made a planned decision before the first gamble took place and a final decision after experiencing the outcome of the first gamble.

The expected values of the gambles ranged from -10 points to 50 points in steps of 10, that later were translated into real money (each point equaled 1 cent). Each expected value (EV) was represented with at least two decision problems.[2] For example, for $EV = 20$ one decision problem consisted of two identical gambles, each offering a 50% chance to win 140 points or lose 100 points. A second decision problem for the same EV gave a 50% chance to win 200 points or lose 160 points. The outcome of the first gamble in each decision problem was controlled. The first gamble was won in half the decision problems and lost in the other half. The order of 16 gambles was counterbalanced and replicated twice. Table 1 presents the entire set of the decision problems (including a practice decision problem) and the order of appearance.

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[1]Beyond the demonstration of the gambler's fallacy, we do not know of any research that has studied how subjective probabilities change with experience. This explanation is suggested as it can qualitatively explain the systematic directions of dynamic inconsistency.

[2]The design tried to avoid a situation where DMs might lose money in the experiment. Thus, we avoided highly negative EV decision problems. Instead the design balanced low EV decision problems (including negative EV) and high EV decision problems. Nine of the 16 decision problems involved low EV that ranged between -10 points and 10 points. The other 7 decision problems involved high EV that ranged between 20 points and 50 points. In order to assess individual utility functions, we used a range of seven gain values (80 points to 200 points in steps of 20) and a range of seven loss values (-100 points to -220 points in steps of 20). Presenting equal EV decision problems with different values also allowed testing the effect of gambles' variance on DMs' choices.

Participants

One hundred students taking an introductory course in psychology at Indiana University participated in the experiment. They were recruited by advertisements offering monetary reward of $2–20 depending on the number of points earned in a gambling experiment. They were paid $2 for participating and received an added bonus based on the points accumulated in four decision problems that were sampled at random from the entire set. Each point earned in the experiment equaled $0.01, and the average payoff was $12.25.

Procedure

DMs were told that they were participating in a decision-making experiment in which each decision problem consists of two gambles in a row. They were told that the first gamble in each decision problem would be obligatory and that they would be asked to make two choices regarding the second gamble. One choice would be made before the first gamble took place and another choice after completing the first gamble. Participants were told that one of their two choices would be sampled at random to determine whether the second gamble would take place or not. Instructions were presented onscreen. For each decision problem, participants were presented with a sequence of onscreen dialogues. Note that they did not actually see any decision trees such as the one shown in Figure 1. Instead, all the information was presented textually. Responses were made using the mouse to choose between possible options (e.g. accepting or rejecting the second gamble). The sequence of the onscreen dialogues for each decision problem allowed participants to: (a) review the values and probabilities of the two gambles; and (b) make planned choices for the second gamble. One planned choice was made contingent on winning the first gamble, and another planned choice was made contingent on losing the first gamble. The plan was worded: '*If I win the first gamble, I will take/reject the second gamble*' and '*If I lose the first gamble, I will take/reject the second gamble*'. The sequence continued to (c) playing the first gamble and experiencing its outcome; and was then followed with (d) making a second (final) choice regarding the second gamble. The final choice was simply worded: '*I will take/reject the second gamble*'. At this point one of the two choices (planned or final) was sampled at random to determine whether the second gamble would take place or not. Each decision problem ended with a summary of the points won or lost in that problem. The first decision problem served as practice to make sure that the participants understood the task. This practice problem was followed with 32 decision problems.[3] Four of the 32 decision problems were sampled at random to determine the final take-home pay for each participant.

RESULTS

Choice proportions

Table 1 presents the overall proportions of planned and final choices to take the second gamble. The first column indicates the order of appearance of each decision problem. The second column provides the gain and loss values of the two identical gambles of the decision problem. The next two columns present the EV and the standard deviation of each gamble. The column titled 'Planned Acceptance' indicates the proportions (in percentage terms) of planned choices to accept the second gamble under anticipated gain and under anticipated loss in the first gamble. The average of planned acceptance was 60% for anticipated gain, and 63% for anticipated loss. A paired t-test was conducted on the logit transformations of the individual proportions, indicating the difference was not significant ($t[99] = 0.014, p < 0.989$). The proportions of planned acceptance are somewhat high. For example, planned acceptance for gambles that have EV of 0 is higher than 50% (between 56% and 64%). The lack of loss aversion in planned choices may be attributed to the demand characteristic of the sequential gambling paradigm. Since the first gamble was obligatory, DMs were put in a risky context that

[3]The findings indicate that there was no significant difference between choices in the practice decision problem and all the other decision problems. Thus, all 33 decision problems were included in the analysis.

Table 1. Proportions of Planned and Final choices to accept the second gamble for each decision problem

				Planned acceptance (shown as %)		Final acceptance (shown as %)	
Order of appearance	Gamble values	EV	STD	Anticipated gain	Anticipated loss	Actual gain	Predicted by ref change
10	200, −220	−10	210	46%	46%	34%	38%
5	180, −200	−10	190	42	47	35	37
2	200, −200	0	200	56	61	51	43
4	120, −100	10	120	68	73	62	52
14	140, −100	20	120	59	64	54	59
15	200, −140	30	170	60	65	53	61
6	200, −120	40	160	74	73	68	68
13	200, −100	50	150	78	79	70	78
				Anticipated gain	Anticipated loss	Actual loss	Predicted by ref change
16	80, −100	−10	90	32	40	44	50
8	100, −120	−10	110	41	53	63	51
17	100, −100	0	100	64	62	64	60
12	200, −180	10	190	57	56	69	63
3	160, −140	10	200	69	68	69	65
9	200, −160	20	180	64	69	72	68
11	160, −100	30	130	66	64	73	73
7	180, −100	40	140	66	70	80	76
1	200, −100	50	150	85	85	82	69

*The proportions are based on 200 observations for each gamble (except for gamble 1—the practice problem—for which there were 100 observations).

may have encouraged further risk taking. However, the correlations between the EV of the gamble and planned acceptance show that DMs' choices took this rational characteristic into account ($r = 0.85$ for anticipated gain, $r = 0.86$ for anticipated loss). The correlations between the gambles' standard deviations and planned acceptance were close to zero ($r = 0.08$ for anticipated gain and -0.1 for anticipated loss).[4]

The column titled 'Actual Acceptance' indicates the proportions (in percentage terms) of final choices to accept the second gamble after experiencing the outcome of the first gamble. The last column of Table 1 indicates the final proportions predicted by the reference-change model and will be discussed later. The upper half of Table 1 shows the decision problems in which the first gamble was won. Note that when DMs experienced gain, the probability of final acceptance was always lower than the probability of planned acceptance (average acceptance was 67%, indicating a decrease of 7%). A paired t-test was conducted on the logit transformations of individual planned and final acceptance proportions. The analysis indicated that the decrease was significant ($t[99] = 4.64$, $p < 0.001$). The lower half of Table 1 shows the decision problems in which the first gamble was lost. When DMs experienced loss, the probability of final acceptance was higher than the probability of planned acceptance, except for one case in the practice problem (average acceptance was 57.6%, indicating an increase of 5.4%). A paired t-test was conducted on the logit transformations of individual planned and final acceptance proportions. The analysis indicated that the increase was significant ($t[99] = -4.06$, $p < 0.000$).

[4]The gambles' variance did not have an effect at the aggregate level. However, it was also tested at the individual level. Later in this paper we present a general model for dynamic inconsistency. This model was tested in a few versions with and without the variance of the gambles (using the same number of parameters). Including the variance improved the model's fit. In fact, Decision Field Theory (which is used as one component of the suggested model) uses the variance term to explain robust violations of independence from irrelevant alternatives (the Myers effect) reported in Busemeyer and Townsend (1993). But this is not the main focus of the paper, and it should not sidetrack the main point.

Table 2. Dynamic consistency and dynamic inconsistency—joint proportions of planned and final choices across trials (given in percentages), average number of cases and standard deviations

		Dynamically consistent		Dynamically inconsistent		
		Consistent acceptance (Ptt)	Consistent rejection (Pnn)	Risk-aversion reversal (Ptn)	Risk-seeking reversal (Pnt)	Total
Actual gain	Percentage	48%	35%	12%	5%	100%
	Average no. cases	7.64	5.56	1.98	0.82	16
	Std	4.79	4.76	2.05	1.10	
Actual loss	Percentage	54%	25%	7%	14%	100%
	Average no. cases	9.13	4.40	1.17	2.30	17
	Std	4.62	4.26	1.48	2.13	

The DMs' ability to predict these systematic changes in preferences was limited. On average, planned acceptance under anticipated loss was somewhat higher than planned acceptance under anticipated gain. As noted earlier, this difference was not significant. Moreover, this difference was not systematic across all decision problems. On 10 of the 17 decision problems, DMs' choices predicted they would be more willing to accept the second gamble after losing the first one than after winning it. For the other 7 decision problems DMs' choices predicted either the opposite pattern or equal proportions.

Dynamic inconsistency

The choice proportions do not necessarily reflect the individual choice pattern and its consistency. We now turn to examine the data at the individual level and present the findings regarding dynamic inconsistency. For each decision problem (i.e. trial) the choices of each DM were recorded as either dynamically consistent or inconsistent. Choices were recorded as dynamically consistent when planned and final choices were identical. One consistent case was when both planned and final choices were to take the second gamble (denoted Ptake-take or Ptt). Another consistent case was when both planned and final choices were to reject the second gamble (denoted Pnot-not or Pnn). Choices were recorded as dynamically inconsistent when the DM's planned choice differed from his/her final choice. Table 2 presents the joint proportions of consistent and inconsistent cases, as well as the average number of cases in each condition, and the standard deviations. As can be seen, the overall proportion of dynamically inconsistent choices was 0.19. When the first gamble was won, the proportion of risk-aversion inconsistencies (planned acceptance and final rejection denoted Ptake-not or Ptn) was 0.12 (an average of 1.98 reversals). The proportion of the risk-seeking inconsistencies (planned rejection and final acceptance denoted Pnot-take or Pnt) was only 0.05 (an average of 0.82 reversals). A paired t-test was conducted to compare the number of risk-aversion reversals and risk-seeking reversals for each DM, indicating the difference of 1.16 was significant ($t[99] = 4.84$, $p < 0.001$).

When the first gamble was lost, dynamically inconsistent choices indicated an opposite pattern. The proportion of risk-aversion inconsistencies, Ptn, was 0.07 (an average of 1.17 reversals), and the proportion of risk-seeking inconsistencies, Pnt, was 0.14 (an average of 2.3 reversals). A paired t-test was conducted to compare the number of risk-aversion reversals and risk-seeking reversals for each DM, indicating that the difference of -1.13 was significant ($t[99] = -4.20$, $p < 0.001$). These findings replicate Barkan and Busemeyer's (1999) earlier findings regarding both the overall proportion of dynamically inconsistent trials and the systematic directions of preference reversals based on the experienced outcome.

MODELING DYNAMIC INCONSISTENCY

The three explanations for dynamic inconsistency are rigorously tested for the first time by formal model comparisons, performed at the individual level of analysis. First, we present a general model for the

Table 3. A schematic representation of the general model: Planned and Final choices and joint choices

Final choice	Planned choice: PG	PnotG	
FG	Pr(PG & FG) *consistent*	Pr(PnotG & FG) *inconsistent*	Pr(FG)
FnotG	Pr(PG & FnotG) *inconsistent*	Pr(PnotG & FnotG) *consistent*	Pr(FnotG)
	Pr(PG)	Pr(PnotG)	

Note: PG stands for planned choice to accept the second gamble. PnotG stands for planned choice to reject the second gamble. FG stand for final choice to accept the second gamble. FnotG stands for final choice to reject the second gamble.

two-stage choices; second, we present a probabilistic model that describes the choice process within each stage, and third, we incorporate different assumptions into the subjective probabilities and utilities to represent the three alternative explanations. Then the parameters of all three models are estimated by maximum likelihood methods separately for each participant's data; and finally, the models are compared using chi-square lack of fit statistics.

General model

The general model describes the joint probabilities of the four possible pairs that can be obtained from the planned and final decisions (see Table 3). The symbol *PG* denotes the choice 'Plan to accept the Gamble', *PnotG* denotes 'Plan to reject the Gamble' (i.e. plan to reject the gamble and win or lose nothing in the second stage). *FG* denotes 'Final choice to accept the Gamble', and *FnotG* denotes 'Final choice to reject the Gamble' (i.e. reject the gamble and win or lose nothing in the second stage). The symbols inside the cells represent the joint probabilities. For example, *Pr(PG&F*notG) represents the joint probability of a planned choice to take the gamble and a final choice to take the certain outcome. The marginal probability of planning to choose the gamble is symbolized as *Pr(PG)*, and likewise, *Pr(FnotG)* denotes the marginal probability for finally taking the sure thing of no-gain no-loss.

The general model assumes that the final decision can be made by one of two strategies: One is to simply recall and repeat the planned choice, and the other is to leave the plan aside and make a new independent choice at the final stage.[5] The probability of repeating the previous choice is represented by a parameter denoted m. On the basis of this general model, the joint probabilities are given by

$$Pr(PG\&FG) = Pr(PG) \cdot [m + (1 - m)Pr(FG)] \tag{1a}$$

$$Pr(PG\&FnotG) = Pr(PG) \cdot (1 - m)Pr(FnotG) \tag{1b}$$

$$Pr(PnotG\&FG) = Pr(PnotG) \cdot (1 - m)Pr(FG) \tag{1c}$$

$$Pr(PnotG\&FnotG) = Pr(PnotG) \cdot [m + (1 - m)Pr(FnotG)] \tag{1d}$$

[5]Leaving the plan aside can occur when the DM forgets what it was or when the DM makes a deliberate decision to make a new choice. We do not distinguish between the two cases and model both with the parameter m, which represents the probability of repeating the planned choice.

If $m = 1$ (i.e. the DM recalls the previous choice and chooses to repeat it) the final choice would be identical to the planned choice and there would be no inconsistent cases. If $m = 0$ (i.e. the DM does not recall or abandons the previous choice), the final choice is independent of the plan and possibly inconsistent. When $0 < m < 1$ it induces some dependence, yet allows some degree of inconsistency. In order to predict the joint probabilities we need to estimate m as well as the marginal probabilities $Pr(PG)$, $Pr(FG)$.[6]

A simplified version of Decision Field Theory (DFT; Busemeyer & Townsend, 1993) was used to predict the marginal choice probabilities, $Pr(PG)$, $Pr(FG)$. This version of Decision Field Theory describes the choice as a process in which a preference evolves to exceed some threshold θ. The preference is determined by the ratio between the difference between the utility of the gamble and the utility of the alternative certain outcome (termed the valence difference or d) and the risk involved in the gamble (i.e. the gamble's variance σ^2). According to this model the probability of choosing the gamble over a sure thing is given by

$$Pr(G) = 1/\{1 + \exp[-2 \cdot \theta \cdot (d/\sigma^2)]\} \tag{2}$$

where $d = \mu - u(C)$ is the difference between the subjective expected utility (μ) of the gamble and the utility of the alternative certain outcome ($u(C)$). The subjective expected utility of the gamble is defined as, $\mu = p \cdot u(g) + (1 - p) \cdot u(l)$, that is, the utilities of the gain and loss, $u(g)$ and $u(l)$, weighted by their subjective probabilities, p and $(1 - p)$. The variance of the gamble, $\sigma^2 = p \cdot u(g)^2 + (1 - p)u(l)^2 - \mu^2$, represents the risk involved in the gamble—the larger the variance, the riskier the gamble. The parameter, θ, represents the threshold bound for making a decision, which is an important determinant of decision time (not measured in the present study).

Baseline model

The Baseline model represents Barkan and Busemeyer's (1999) first explanation. Recall that according to this explanation the inconsistencies between planned and final choices reflect mere choice inconsistency that is due to random fluctuations in preference. The random fluctuations are represented with the parameter m. In order to make this model a critical contender, we incorporate in it a Prospect Theory-like utility function. This model uses equation (2) to predict the marginal probabilities $Pr(PG)$ and $Pr(FG)$. It then uses equation (1) to predict the joint probabilities.

The Baseline model adds the following assumptions to equation (2). First, the gambles' values (i.e. points) are transformed to utility units. The utility function of the DM is represented by a two-part power function (cf. Tversky & Shafir, 1992): the utility of a gain value is $u(g) = g^\alpha$, where α is the exponent for gains; the utility of a loss value is $u(l) = -|l|^\beta)$, where β is the exponent for losses. In line with Prospect Theory the two parameters α and β allow an asymmetric utility function (Tversky & Kahneman, 1992; Tversky & Shafir, 1992).[7] Second, according to the Baseline model, the planned choice and the final choice differ

[6]The estimations of $Pr(PG)$, $Pr(FG)$ serve as a basis to predict the joint probabilities which are at the focus of the present study. Thus, we do not explicitly present the estimates of the marginal probabilities.

[7]The representation of the utility function follows Tversky and Shafir (1992) and uses different exponents for the gain domain and the loss domain. Another common representation of the utility function uses only one exponent for both domains and an added loss aversion parameter λ for the utility of the loss domain. According to this representation the utility of a gain is $u(g) = g^\alpha$ and the utility of a loss is $u(l) = -\lambda(-l)^\alpha$ (Benartzi & Thaler, 1995; Tversky & Kahneman, 1991). In addition to the computations reported in this paper, we computed predictions of the Reference-change model using this representation of the utility. The Reference-change model with one exponent and loss aversion parameter produced the same qualitative predictions as the Reference-change model with different exponents for gains and losses. The mean parameters of the utility function under this model were $\alpha = 0.94$, and $\lambda = 1.31$. The correlation between this model's prediction and final acceptance of the second gamble was $r = 0.89$. The quantitative fits for the individual DMs were slightly better than those provided by the Reference-change model with two different exponents (the average Chi-square difference was 2.68 $t[99] = 4.18, p < 0.0001$). Interestingly, the advantage of this model was due to extremely good fits for 8 subjects that showed completely consistent behavior. Since the minor advantage of this model was not derived from better modeling of dynamic inconsistency, it was declined. Further research is needed to compare the difference between the two definitions of the utility functions.

due to m (i.e. random fluctuation), but not due to a change in the reference point or in the subjective probabilities. In order to represent the fixed reference point, we assume that choosing the sure thing implies a certain outcome of zero (i.e. $u(C) = 0$).[8] To represent the lack of change in the subjective probabilities, we assume that the subjective probabilities coincide with the stated probability ($p = 0.50$ in this case).[9]

In sum, the Baseline model has four free parameters θ, m, α, β. Note that according to this model, *Pr(PG&FnotG)* = *Pr(PnotG&FG)* is true for any set of parameter values. That is, the probabilities of different directions of inconsistencies are equal. Thus, this model fails to explain the observed pattern of dynamic inconsistency. It does, however, provide a baseline that the next two explanations must exceed to be considered viable alternatives.

Reference-change model

The Reference-change model follows the second explanation. Recall that according to this explanation, experiencing the outcome of the first gamble changes the utility associated with the second gamble. The neutral reference point used for the planned choice reflects an isolated evaluation of the utility of the second gamble. The changed reference point used for the final choice reflects an integrated evaluation of the utility of the second gamble. For the planned choice, this model uses the same probabilities and utilities as the Baseline model. The probability of choosing the gamble during the planning stage is exactly the same for the Reference-change and Baseline models. Using the terms of Kahneman and Tversky (1979), the evaluation of the utility of the second gamble during planning is made in isolation. For the final choice, the probabilities also remain the same as the Baseline model (i.e. $p = 0.50$), but now we allow the experienced outcome to change the reference point of the utility function as follows. Define the experienced outcome of the first gamble as r. If the first gamble was won, the new reference point would be shifted down (i.e. if $r > 0$, and the utility of the sure thing is $u(C) = r^{\alpha}$). If the first gamble was lost, the new reference point would be shifted up (i.e. if $r < 0$, and the utility of the sure thing is $u(C) = -|r|^{\beta}$). Integrating r to the possible outcomes of the second gamble also changes its subjective expected utility (μ). The two redefined outcomes are $g + r$ and $l + r$. If either of these redefined outcomes is positive, its utilities would be determined using α (i.e. if $g + r > 0$, $u(g + r) = (g + r)^{\alpha}$, if $l + r > 0$, $u(l + r) = (l + r)^{\alpha}$). If either of the redefined outcomes is negative, its utilities would be determined using β (i.e. if $g + r < 0$, $u(g + r) = -|g + r|^{\beta}$, if $l + r < 0$, $u(l + r) = -|l + r|^{\beta}$ (see also footnote 7). Using the terms of Kahneman and Tversky (1979), the re-evaluation of the utility of the second gamble uses the integrated form of the gamble (contingent on the experienced outcome).

In sum, this model has the same number of free parameters (θ, m, α, β) as the Baseline model. However, unlike the Baseline model, this model is forced to predict that *P(PG&FnotG)* $\neq$ *P(PnotG&FG)*. This feature may be an advantage or a disadvantage, depending on whether the predicted differences are in the correct direction. Thus the Reference-change model may perform better or worse than the Baseline model.

Probability-change model

The Probability-change model follows the third explanation. According to this explanation, experiencing the outcome of the first gamble changes the subjective probability associated with the second gamble. For the planned choice, this model uses the same probabilities and utilities as the Baseline and Reference-change

[8]The neutral reference point used for both the planned and final choice ($u(C) = 0$ in the Baseline model) is consistent with the assumption that the reference point is always neutral and that its experienced utility is zero.
[9]The non-linear weighting of probabilities suggested by Prospect Theory is not utilized for the present modeling. This element is less critical in the current setting as all the gambles involve equal probabilities of 0.50. That is the weight $\pi(0.50)$ would be constant for all the gambles. However, nonlinear weighting may be significant in more complex cases, where the probabilities are unequal or extreme.

models. Note that the probability of choosing the gamble during the planning stage is exactly the same for all three models. For the final decision, we now allow the subjective probabilities to be affected by the outcome experienced during the first stage. Following a gain, the subjective probability of winning again is represented by a free parameter, denoted ρ. The complementary subjective probability $1 - \rho$ is associated with winning the second gamble after experiencing a loss in the first gamble. According to this model, the utility of the gain and loss ($u(g)$, $u(l)$) and the utility of the certain outcome ($u(C) = 0$) are not affected by the experienced outcome.[10] Furthermore, to equate the number of free parameters, we use the same exponent for both parts of the utility function ($u(g) = g^{\alpha}$, and $u(l) = -|l|^{\alpha}$).[11]

In sum, this model has the same number of free parameters (θ, m, α, and ρ) as the Baseline and Reference change models. However, unlike the previous two models, this model may or may not predict that *P(PG&FnotG)* = *P(PnotG&FG)*. Note that the specific prediction depends on specific parameter values for the subjective probabilities. Thus the Probability-change model may perform better or worse than either the Baseline model or the Reference-change model.

Parameter estimation

An important contribution of the present work is an examination of the three explanations at an individual level of analysis. This is a new and important technology for decision research that avoids the pitfalls of evaluating models based on average data. Theories of decision making are stated for individuals, and allow individual differences in parameters. Averaging across these individual differences can lead to invalid conclusions (Coombs et al., 1970).

The four free parameters of each model were estimated separately for each of the 100 participants using the following procedure. The DM's joint choices for each decision problem (i.e. trial) were recorded with four binary valued variables: $Xtt_{(t)} = 1$, whenever a DM consistently chose the gamble at both stages of a trial; $Xnn_{(t)} = 1$, when a DM consistently rejected the gamble at both stages of a trial; $Xtn_{(t)} = 1$, when a DM planned to choose the gamble, but finally rejected the gamble in trial; and $Xnt_{(t)} = 1$, when a DM planned to reject and finally chose the gamble in a trial. In each trial only one of these variables was recorded as 1 (i.e. the observed pattern), and the other three were recorded as zeros. We ended up with a matrix for each DM. This matrix $\mathbf{X}_{(t)} = [Xtt_{(t)}, Xnn_{(t)}, Xtn_{(t)}, Xnt_{(t)}]$ had 33 rows, each representing the DM's joint choices in each decision problem.

Each model's predictions followed equations (1) and (2) (with the specific parameters described above), and provided four probabilities for each decision problem. The vector of predicted probabilities $\mathbf{P}_{(t)} = [Ptt_{(t)}, Pnn_{(t)}, Ptn_{(t)}, Pnt_{(t)}]$ corresponded to the vector of the four observed variables $\mathbf{X}_{(t)} = [Xtt_{(t)}, Xnn_{(t)}, Xtn_{(t)}, Xnt_{(t)}]$. The parameters were selected to come up with a predicted pattern that would be as close as possible

[10]Note, however, that the expected utility of the gamble is affected by the change in the subjective probability. If the first gamble was won, the expected utility of the second gamble at the final stage would be $\mu = \rho \cdot u(g) + (1 - \rho) \cdot u(l)$. If the first gamble was lost, the expected utility of the second gamble at the final stage would be $\mu = (1 - \rho) \cdot u(g) + \rho \cdot u(l)$.

[11]We also examined a version of the probability change model that allowed an asymmetric utility function (i.e. different exponents for gains and losses). Another option was to allow two subjective probability parameters, one for experienced gain and another for experienced loss. Both alternatives employ 5 parameters, whereas all of the other models considered here have only 4 parameters. To compare non-nested models that differ in terms of the number of parameters, we employ the Bayesian Information Criterion, or BIC (see Wasserman, 2000, for more discussion of this model comparison criterion). This criterion is designed to select the model that has the highest probability of being correct given the observed data. According to this criterion, BIC $= \chi^2 + k\ln(N)$, where k is the number of parameters, and N is the number of observations. In other words, the BIC equals the chi-square lack of fit plus a penalty for the number of parameters. The model producing the smallest BIC corresponds to the model that is more likely to be correct given the data. In this case, $k = 5$ for the probability change model and $k = 4$ for the reference-point model, and $N = 33$. Using this criterion, the BIC(5 parameter Probability-change model)—BIC(4 parameter Reference-change model) $= 1.94 > 0$, indicating that four-parameter Reference-change model has a higher probability of being correct than the five-parameter Probability-change model. A similar conclusion is reached using an alternative model comparison criterion, the Akaike Information Criterion (AIC).

to the observed pattern. That is, the parameters were selected to maximize log likelihood for each DM (i.e. minimizing the chi-square lack of fit index).[12]

$$\chi^2 = -2 \cdot \Sigma_{t=1,33}\{Xtt_{(t)} \cdot \ln[Ptt_{(t)}] + Xnn_{(t)} \cdot \ln[Pnn_{(t)}] + Xtn_{(t)} \cdot \ln[Ptn_{(t)}] + Xnt_{(t)} \cdot \ln[Pnt_{(t)}]\}$$

The parameter estimation procedure is illustrated with the following hypothetical simple case. Suppose a DM was presented with just one decision problem. Suppose the DM planned to take the second gamble if the first gamble was won, but after experiencing a gain in the first gamble the DM reversed the decision and chose not to take the second gamble. The binary pattern for this decision problem would be 0 0 1 0 (corresponding to *Xtt*, *Xnn*, *Xtn*, *Xnt*). Any values for θ, m, α, β (for the Baseline and Reference-change model) and θ, m, α, ρ (for the Probability-change model) would result in corresponding predicted probabilities. The predicted probabilities cannot give a binary pattern of zeros and one. However, we require that they would be as close as possible to this pattern. The estimation procedure would search for the best parameter values. For this hypothetical case suppose that one set of parameters (for one of the three models) resulted in the corresponding predicted probabilities: 0.026 0.071 0.821 0.082. Note that the highest probability is given to *Ptn*; thus the predicted pattern corresponds to the observed one. The chi-square lack of fit index measures the discrepancy between the observed and predicted pattern. In this hypothetical example it would be:

$$\chi^2 = -2 \cdot \{0 \cdot \ln[0.026] + 0 \cdot \ln[0.071] + 1 \cdot \ln[0.821] + 0 \cdot \ln[0.082]\} = -2\{-0.197\} = 0.394$$

The parameter estimation procedure searches for parameter values that minimize the measure of χ^2 lack of fit. In the present work, each DM was presented with 33 decision problems (1 practice problem and two replications of the set of 16 decision problems). The parameter estimation procedure was required to find one set of parameters' values that provides the best fit for all the 33 binary patterns of choice consistency for each DM.

Model comparisons

The predictions provided by the three alternative models are shown in Figure 3. All three models capture the general rate of inconsistency (i.e. $Ptn + Pnt$). However, for dynamic inconsistency, it is crucial to explain the direction of inconsistency (i.e, *Ptn-Pnt*) and its dependence on the experienced outcome. As noted earlier, the Baseline model cannot capture this pattern because it predicts equal rates for the two types of inconsistencies (i.e. $Ptn = Pnt$), regardless of the experienced outcome. Both the Reference-change and the Probability-change models captured the directions (and magnitudes) of dynamic inconsistency (i.e. $Ptn > Pnt$ after experienced gain, and $Ptn < Pnt$ after experienced loss).

Quantitative comparisons between the Baseline, Reference-change, and Probability-change models were based on the models' chi-square scores for each DM. Chi-square differences were computed for each pair of competing models. As can be seen in Table 4, the Reference-change model fit the individual DMs better than the Baseline model for a majority of individuals (67%), and the mean difference was significant ($t[99] = 3.49$, $p = 0.0004$). The Reference-change model was also superior to the Probability-change model for a majority of individuals (68%), and the mean difference was again significant ($t[99] = 4.62, p < 0.0001$). The Probability-change model was no better than the Baseline model (the former was favored for 49% of the DMs), and the mean difference was not significant ($t[99] = -1.44$, ns).

Note that the Probability-change model provided a fairly good fit for the overall pattern at the aggregate level, but failed to fit the individual level. The failure of the Probability-change model is interesting and

[12]Model analyses were computed using Matlab. The Fmins procedure was used to find parameters that maximize log likelihood for each DM (i.e. minimize Chi-square). To facilitate interpretation of the parameters, all gamble values were divided by 200 in the computations.

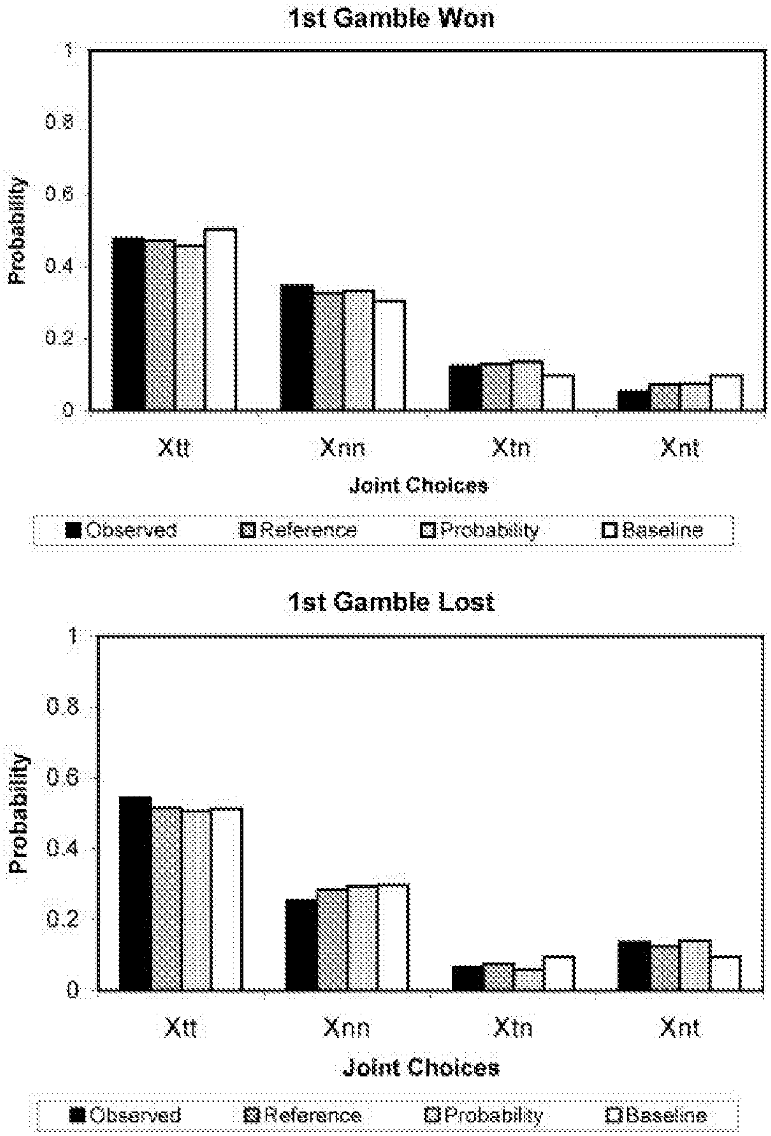

Figure 3. Comparing the experimental findings and the predictions of the three alternative models. The tallest bars at the left (labeled Xtt) represent consistent acceptance of the second gamble. The bars next to them (labeled Xnn) represent the cases of consistent rejection. The bars labeled Xtn represent dynamic inconsistency in the direction of risk aversion. The bars labeled Xnt represent dynamic inconsistency in the direction of risk seeking

points to the importance of individual difference analyses. The reason for this failure is attributed to the symmetry with which this model deals with gains and losses. First, this model uses a symmetric utility function which is unreasonable given the different perceptions of gains and losses.

Another limitation of this model is the simple mechanism that updates the subjective probability parameter, ρ. This mechanism is also symmetric in the following sense. Suppose, for example, that the first gamble was won. The DM would now perceive the subjective probability for another gain in the second gamble as $\rho = 0.50 - \Delta$. The subjective probability for a loss in the second gamble would be $1 - \rho = 0.50 + \Delta$.

Table 4. Model comparisons based on chi-square lack of fit statistics

	Number of DMs won by each model	Mean difference in chi-squares	Std of difference	$T(99)$	p
Baseline versus probability	50–49	−1.09	7.53	−1.44	0.0760
Baseline versus ref. point	33–67	1.69	4.84	3.49	0.0004
Probability versus ref. point	32–68	2.78	6.00	4.62	0.0000

Note: $T(99)$ stands for the statistic for a t-test with 99 degrees of freedom, and p stands for the p-value produced by the corresponding t-statistic.

The decrease in the subjective probability for another gain equals the increase in the subjective probability for the loss. Note that the same Δ would be used if the first gamble was lost. In this case the subjective probability for a second gain would be $\rho = 0.50 + \Delta$, and the subjective probability for a second loss would be $\rho = 0.50 - \Delta$. Even more important, Δ is independent of the magnitude of the gain and loss. Thus, the decrease in the subjective probability for a second gain would be the same whether that gain was of 1 point or 1,000,000 points. The models' comparison indicated that these assumptions are invalid. While the logic of the Probability-change model can capture the overall pattern, further assumptions and more parameters are needed in order to allow sensitivity to different values of gains and losses (see also footnote 11).

In brief, the Reference-change model provided the best explanation of the patterns of choices produced by the individuals (see also footnote 7). Additional support for this model comes from its predictions for the final choice probabilities for each gamble (i.e. $Pr(FG)$). The choice probability predictions for the Reference-change model, averaged across subjects, are shown in the last column of Table 1. The correlation between the EVs of the gambles and DMs' final acceptance of the second gamble was $r = 0.72$. The DMs' plans were a better predictor of their final acceptance (the correlation was $r = 0.84$). The correlation between the Probability-change model's predictions and final acceptance of the second gamble was $r = 0.87$. The highest correlation was found between the predictions of the Reference-change model and final acceptance ($r = 0.90$).

INDIVIDUAL DIFFERENCES

Table 5 indicates the means and standard deviations (and ranges) for the parameters of the Reference-change model. As can be seen, the means of the utility parameters α and β are in line with the utility function suggested by Prospect Theory (Tversky & Kahneman, 1992). The utility function is concave for gains, convex for losses, and the function is steeper for losses than for gains. The mean of the parameter m indicates that DMs repeated their planned choices in approximately half of the trials. Finally, the standard deviations imply considerable individual differences in these parameters.

Individual differences in model parameters were correlated with the general inconsistency rates (i.e. $\Sigma_t Xtn_{(t)} + Xnt_{(t)}$ for each DM). Note that general inconsistency rates ignore the direction of the change in choice and its dependency on the experienced outcome. Two of the model parameters, m and θ, were found

Table 5. The parameters of the reference-change model

	α	β	θ	m
Mean	0.85	0.95	2.80	0.57
std	0.69	0.65	1.78	0.27
Range	0–2	0–2	0–5	0–1

Note: α is the exponent for gains; β is the exponent for losses; θ is the cutoff and m reflects the strategic parameter to repeat the planned choice.

Table 6. Frequencies of different choice patterns by three levels of m

m	Choice pattern		
	Consistent	Random	Dynamic inconsistency
1–0.65	13	9	17
0.64–0.35	0	10	31
0.34–0.00	0	1	19

to be significantly correlated with the general inconsistency rates of the DMs. The negative correlation between m and general inconsistency ($r = -0.88$, $p < 0.001$) indicated that as the tendency to repeat the planned choice decreases, the amount of general inconsistency increases. Thus, not surprisingly, leaving the planned choice aside and coming up with a new choice serves as a prerequisite for general inconsistency. The negative correlation between θ and general inconsistency ($r = -0.39$, $p < 0.001$) indicated that as the threshold decreases, choice becomes less deterministic and more random, and general inconsistency increases.

An individual difference analysis of dynamic inconsistency was also performed. This analysis referred to the direction of inconsistency that is contingent on the experienced outcome (i.e. $Xtn - Xnt$). The 100 DMs were categorized according to their patterned choice behavior (consistent, non-directional inconsistency and directional inconsistency). Table 6 shows the frequencies of the three patterns according to three levels of the m parameter. As can be seen, consistent behavior was observed for only 13 DMs at the highest level of m. As m decreased below 0.65, the number of consistent DMs dropped to zero. Twenty DMs showed a non-directional type or random inconsistency. This pattern dropped markedly as m decreased below 0.35. Sixty-three DMs showed directional inconsistency. While it is expected that consistent decision makers must have high value of m, it is not obvious that lower levels of m should result with dynamically inconsistent decision makers. Lower levels of m can easily result with random inconsistency. The interesting finding in Table 6 is that in each and every level of m, the majority of the decision makers exhibit dynamic inconsistency rather than random inconsistency. The relative frequency of this pattern increased as memory decreased ($\chi^2[4] = 29.34$, $p < 0.001$).

Whereas m is not sufficient to differentiate between random and dynamically inconsistent decision-makers, these two groups can be differentiated according to their utility functions (parameters α and β). Figure 4 gives as an example the median utility functions of non-directional and directional inconsistency groups. Non-directional or randomly inconsistent decision makers exhibit utility functions that are close to linear and dynamically inconsistent decision makers exhibit Prospect Theory-like utility functions. This finding is inline with the Reference-change model's argument that different patterns of choice behavior depend on the individual utility functions (i.e. parameters α and β).

DISCUSSION

The principle of dynamic consistency asserts that preferences should be stable whether they are based on anticipated events or (later on) on the experience of the same events. The current findings indicate that in many cases DMs violate this principle and show preference reversal due to actual experience of the anticipated events on which they based their plans. Preference reversals were found to be systematic and dependent on the nature of the experienced event. Experiencing an anticipated gain caused a risk-aversion reversal, and experiencing loss caused a risk-seeking reversal. These findings replicate and extend the phenomenon of dynamic inconsistency reported earlier by Busemeyer et al. (2000), Barkan and Busemeyer (1999), and Cubitt et al. (1998).

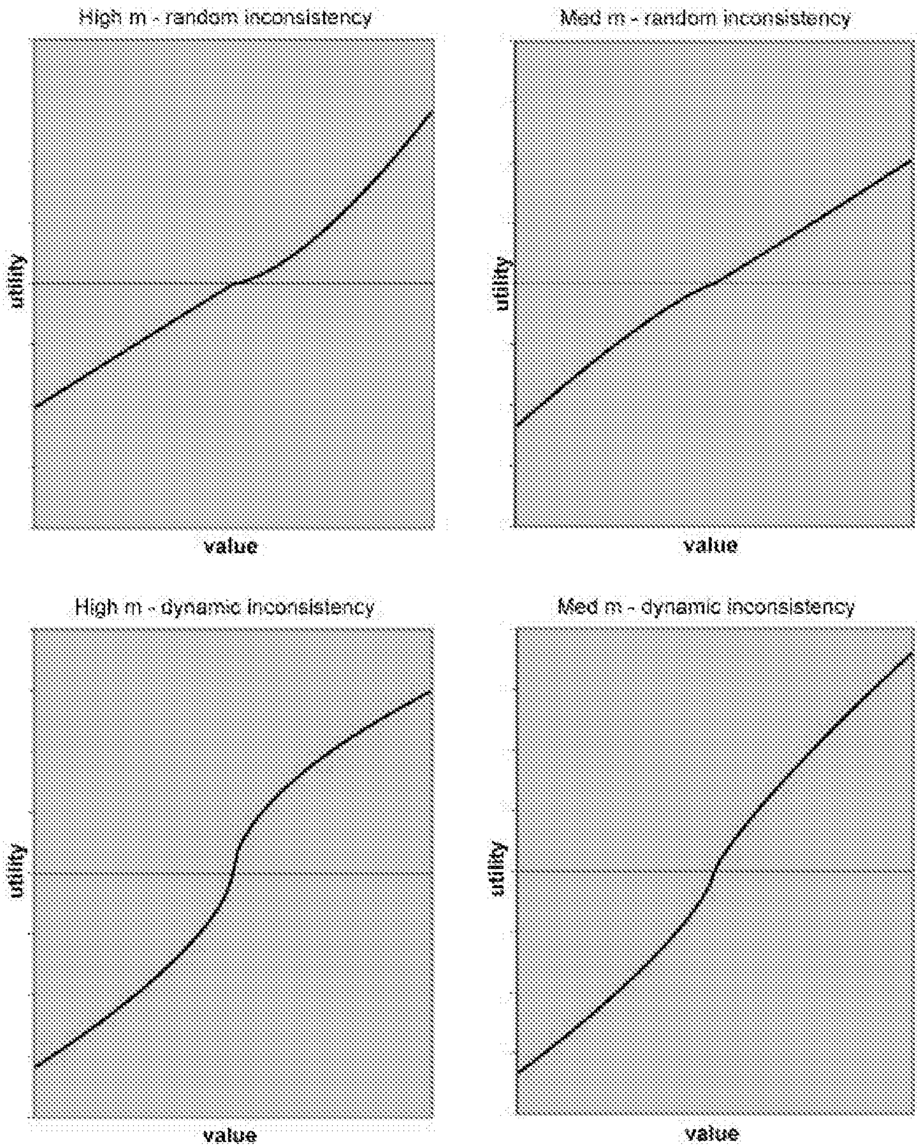

Figure 4. Examples of median utility functions for choice patterns of random inconsistency and dynamic inconsistency. Random inconsistency is characterized with almost linear utility functions. Dynamic inconsistency is characterized with Prospect-Theory-like utility functions

The model comparisons support the Reference-change model over the Probability-change and Baseline models as an explanation for the observed dynamic inconsistency. According to this explanation, the reference point changes as the DM progresses through the planned path in a decision tree. The change of reference point results in a change of the utility associated with the planned choice and leads to preference reversal. Using Kahneman's (1999) terms, the change in the reference-point reflects the difference between the decision utility and the experienced utility. Using Prospect Theory's utility function as a representation of the decision utility, the reference-point is at the inflection of the function and its decision utility is zero. A neutral reference-point serves as the basis for the planned choice (that is, zero payoffs are assigned zero utility). The Reference-change model utilizes the Prospect Theory utility function as a representation for the experienced utility as well. However, in this case, the experienced utility of the reference-point is represented with an upward or downward move (according to the experienced outcome), so that the experienced utility of the reference-point is non-neutral.

The superiority of the Reference-change model over the two alternative models is attributed to two elements. One is the asymmetric utility function it assumes, and the other is the structural assumption that changes the utility of the reference point and the second gamble at the final choice. Together, these elements represent the experienced utility in a way that is sensitive both to the sign of the experienced outcome and to the specific value of the experienced outcome. These elements are also sensitive to the specific values of the next gamble that is considered as well as to the change in its utility.

The modeling procedure employed in this work offers two noteworthy advantages. First, an important contribution of the modeling is in its focus on the individual level. Behavioral phenomena are frequently described and explained at the aggregate level, leaving a question regarding the behavior at the individual level. As a relevant example consider the disjunction effect. Tversky and Shafir's (1992) explanation was given at the aggregate level. They supported their explanation with a hypothetical example of a utility function that could reproduce their findings. The reference point explanation extended their argument to explain dynamic inconsistency. However, applying this explanation at the aggregate level posed a problem. In fact, the Reference-change model and the Probability-change model could not be separated at this level and seemed equally effective at explaining the phenomenon of dynamic inconsistency. Turning to the individual level provided a critical test. The present design allowed sufficient leverage to model the alternative explanations for each DM. At this level of analysis, the Reference-change model was superior to the Probability-change model. Note that the individual utility functions used by the Reference-change model followed the same definition used by Tversky and Shafir (1992). The parameters α and β (i.e. exponents for the gain and loss domains) were fitted for a wide range of values. The resulting individual utility functions were robust enough to explain many different choices of the majority of the individual DMs. Thus, the present work also provides additional support for Tversky and Shafir's suggestion.

The second advantage of the present modeling procedure relates to the model's robustness and sensitivity to employed assumptions. Note that the three models compared in this study were all derived from one general model. The three variations of the general model only differed in terms of the assumptions they employed. The model comparisons allowed us to test the validity of the different assumptions. For example, the Baseline and Reference-change models employ exactly the same choice mechanism and the same parameters. The difference between them is a structural assumption regarding the reference point. In the Baseline model the assumption is that the reference point does not change. In the Reference-change model the assumption is that the reference-point does change. Comparing the two models showed that the assumption regarding the change in the reference point is crucial for reproducing the observed choice behavior. As a second example, consider the comparison between the Reference-change and the Probability-change models. These two models differ in their assumption regarding the shape of the utility function. According to the Reference-change model, the utility function is assumed to be asymmetric for gains and losses, whereas the Probability-change model assumes symmetry. Comparing the two models showed that the assumption regarding asymmetric utility function was not needed on the aggregate level, but was crucial for reproducing the individual choice behavior.

CHANGE OF REFERENCE AND OTHER PREDICTION ERRORS

The literature provides many examples of systematic prediction errors regarding future tastes and preferences (for comprehensive reviews see Kahneman, 1999; Loewenstein & Schkade, 1999). Some of these mis-predictions may be explained with a change in the reference point. The endowment effect (Loewenstein

& Adler, 1995; Kahneman et al., 1991) serves as one example. In a typical experiment DMs were presented with an option of choosing between a mug and a preferred sum of money. DMs asked for a median of $3.50. However, once they had the mug, they were only willing to return it for a median of $7.12 (Kahneman et al., 1991). Kahneman (1999) explained the endowment effect with two reference points. According to this explanation, the choice between the endowed object and money is evaluated against different reference points of 'not having' and 'having' the object. When the reference point is that of 'not having', the mug is evaluated as an object soon to be gained. When the reference point is that of 'having', the mug is evaluated as an object soon to be lost. DMs compensate for the experience of loss by asking for more money.

Mis-predictions of future preferences were also found for unpleasant or painful experiences. For example, Christensen-Szalanski (1984) found that a majority of expectant women stated a preference not to have anesthesia during childbirth, but reversed their prior decisions when they went into labor. Before going into labor, the preference not to have anesthesia may be based on a neutral reference point (i.e. no pain). However, the experience of pain during childbirth shifts the reference point to the loss domain. The option of having anesthesia may now seem more attractive than before, and lead to the preference reversal.

Predicting the pain during childbirth may be difficult since it is not an everyday experience and the choice involves other factors such as beliefs and social norms. Other studies show that DMs have difficulty in predicting how experience would change their reference points even when choices are simple and routine. Kahneman and Snell (1992) provide one example. In their study DMs predicted that eating ice cream or yogurt for eight successive days would decrease their liking of the former and increase their dislike of the latter. This prediction corresponds to DMs' expectation that experience would shift the reference point up. However, the correlations between subjects' predictions and their actual ratings of liking were close to zero. While subjects' tastes did change over time, each DM could not predict the exact change in his or her own taste. In a similar fashion, Read and Loewenstein (1995) let students choose snacks for three successive class sessions. One group had to plan ahead of time and choose all the snacks at once, while another group made one choice in each class session. The group that planned ahead of time showed a larger variety in their choices. Such planning corresponds to DMs' predicting that the reference for the same snack would shift up and that their liking for that snack would decrease. However, students in this group were less satisfied with their choices, and later regretted changing snacks.

Loewenstein and Schkade (1999) suggest that planned choices are made in a 'cold' or 'rational' state, whereas choices made during the actual experience reflect a 'hot' or 'emotional' state. Kahneman et al. (1997) suggest a similar differentiation between decision utility and experienced utility. The Reference-change model offers a possible quantification of these concepts. The decision utility or the 'cold' state in which plans are made is represented with a neutral reference point. In contrast, the experienced utility or the 'hot' states in which evaluations are based on immediate hedonic and affective experience are represented with a shifted reference point.

UNDERSTANDING THE DYNAMICS OF THE REFERENCE POINT

The importance of the reference point has long been recognized in the literature on dynamic decision making. It is unclear, however, whether and how the reference point changes in the process of repeated choices. One option is that in order to minimize the cognitive effort of the evaluation process, each event in the dynamic process is coded separately (Tversky & Kahneman, 1981). In this case, the reference point should be neutral for each of the evaluations. Another option suggests shifts in the reference point during the dynamic process: '. . . A discrepancy between the reference point and the current asset position may . . . arise *arisebecauseofrecentchangesinwealthtowhichonehasnotyetadapted′(KahnemanTversky*, 1979, *p*.286).

Thaler and Johnson (1990) tested a number of editing rules representing different assumptions regarding the reference point and the way it shifts due to prior gains and losses. The main difference between the editing rules has to do with the question of whether prior gains and losses are integrated or segregated from subsequent outcomes. For example, assuming the asymmetric utility function of Prospect Theory, integration of prior gains would be more likely to lead to risk aversion, and segregation of prior gains would be more likely to lead to risk seeking. On the other hand, integration of prior losses would be more likely to lead to subsequent risk seeking, while segregation of prior losses would be more likely to lead to subsequent risk aversion. The logic of integration and segregation explains the 'house money effect' and 'break-even effect' in which both prior gains and prior losses lead to risk seeking (respectively).

The findings reported here showed a variety of responses to prior or experienced outcomes. Consistent trials (or DMs) seem to make the case as if prior or experienced outcomes do not affect preferences. Dynamically inconsistent trials (or DMs) suggest the opposite. The frequent effects of experienced outcome were risk aversion after experienced gain and risk seeking after experienced loss, suggesting integration of prior outcomes. However, we also found small measures of risk seeking after experienced gain (as in the 'house money effect') and risk aversion after experienced loss, suggesting segregation of prior outcomes. It is somewhat unclear whether these findings point to a reliable (yet small) effect, or at random behavior that should be considered as noise. Future research is needed to address this question. Such research may utilize manipulations that encourage 'house money' reversals in preferences.

Interestingly, the Reference-change model was able to reproduce all of these contradictory effects (see Figure 3). To account for the different effects the model always utilized the same mechanism of integrating the experienced outcome in the final evaluation but not in the planned evaluation. Different effects of the experienced outcome depended on the individual utility functions of the DMs. Thus, changing the reference point could lead to consistent or inconsistent preferences based on the change in the utility of the integrated outcomes. While such a mechanism seems appealing in its parsimony, future research and further individual level analyses are needed in order to determine the changing mechanism of the reference point.

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