Computational models of decision making

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Abstract

This chapter presents a connectionist or artificial neural network approach to decision making. An essential idea of this approach is that decisions are based on the accumulation of the affective evaluations produced by each action until a threshold criterion is reached. This type of sequential sampling process forms the basis for decision making in a wide variety of other cognitive tasks such as perception, categorization, and memory. We apply these concepts to several important preferential choice phenomena, including similarity effects, attraction effects, compromise effects, loss aversion effects, and preference reversals. These analyses indicate that a relatively complex model of an individual's choice process reveals a relatively simple representation of the individual's underlying value structure.

What are computational models of cognition?

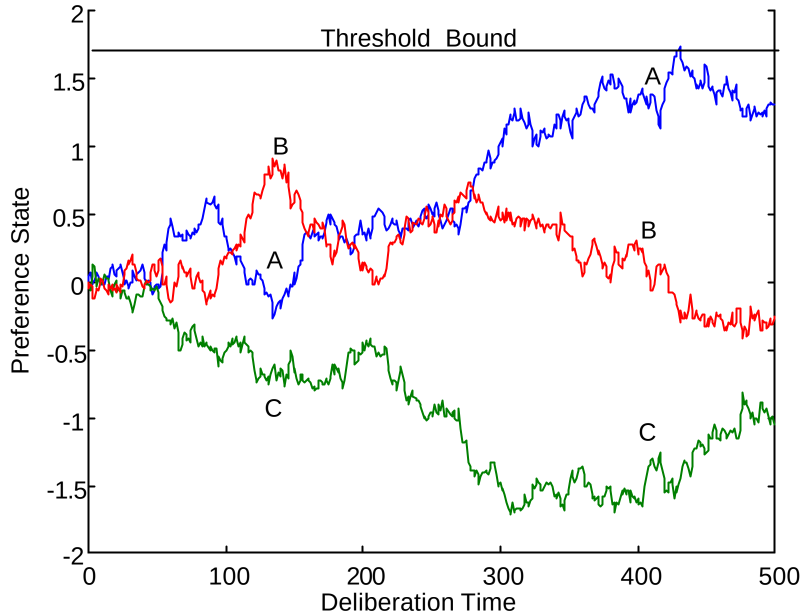
In his classic book on computational vision, Marr (1982) proposed three levels of theories about cognitive systems. At the highest level, theories aim to understand the abstract goals a system is trying to achieve; at an intermediate level, theories are designed to explain the dynamic processes used to achieve the top level goals; and at the bottom level, theories attempt to describe the neurophysiologic substrate of the second level. Judgment and decision-making researchers have generally been concerned with theorizing at the higher and more abstract levels. From this higher point of view, explanations based on principles such as context dependent weights, loss aversion, and anchoring-adjustment are considered satisfactory. This chapter presents arguments for viewing decision making from the perspective of a lower level microanalysis. By doing so, we can try to answer deeper questions such as: why decision weights change across contexts, why people are loss averse, and why anchors are more influential than adjustments.

Computational models are constructed from simple units that conform to a small number of elementary principles of cognition, but a large number of these simple units are connected together to form a dynamical system. Although the properties of the individual units are simple, the emergent behavior of the ensemble becomes fairly complex. Computational models appear in a variety of forms, but this chapter focuses on a class known as artificial neural networks, connectionist networks, or parallel distributed processing systems (see Grossberg, 1988; and Rumelhart & McClelland, 1986, for general overviews of these models). This class of computational models is designed to form a bridge that mediates between the neural and behavioral sciences.

How does the brain make decisions?

A decade ago, the brain was an impenetrable black box, but with recent advances in neuroscience, we can start to look inside. It is informative to point out a conclusion arising from converging evidence obtained through neuroscience research on decision-making. Neuroscientists have examined decision-making processes in the brains of Macaque monkeys using single cell recording techniques (for reviews, see Gold & Shadlen, 2001, 2002; Platt, 2002; Schall, 2001), as well as from the brains of humans using evoked response potentials (Gratten, Coles, Sirevaag, & Donchin, 1988). A simple but important conclusion from this work is that decisions in the brain are based on the dynamic accumulation of noisy activation for each action, and the action whose activation first exceeds the threshold is chosen. This process is illustrated in Figure 1, for three actions, with each trajectory representing the cumulative activation (i.e., preference state) for an action. The horizontal axis represents deliberation time and the vertical axis indicates the activation for each action at each moment in time. In this figure, action A reaches the threshold first, and is chosen at time T = 425.

Figure 1: The decision process for a choice among three actions



This dynamic decision process is known as a sequential sampling process (DeGroot, 1970). It forms the basis of decision models used in a wide variety of cognitive applications including sensory detection (Smith, 1995), perceptual discrimination (Laming, 1968; Link & Heath, 1975; Usher & McClelland, 2001; Vickers, 1979), memory recognition (Ratcliff, 1978); categorization (Nosofsky & Palmeri, 1997; Ashby, 2000), probabilistic inference (Wallsten & Barton, 1982) and preferential choice (Aschenbrenner, Albert, & Schmalhofer, 1984; Busemeyer, 1985).

Computational models for Decision Making.

Several artificial neural network or connectionist models have been recently developed for judgment and decision tasks: some placing more emphasis on the neural processing aspects (Grossberg & Gutowski, 1987; Levin & Levine, 1996; Usher & McClelland, 2002), and others placing more emphasis on applications to judgment and decision making (Holyoak & Simon, 1999; Guo and Holyoak, 2002; Busemeyer &

Townsend, 1993; Roe Busemeyer, & Townsend, 2001). Here we will initially focus on our own, known as decision field theory, but we will also compare this to others later in this chapter.

Decision field theory uses a sequential sampling process to make decisions, consistent with the other areas of cognition. This theory has been applied to a variety of traditional decision making problems including decision making under uncertainty (Busemeyer & Townsend, 1993), selling prices and certainty equivalents (Busemeyer & Goldstein, 1992; Townsend & Busemeyer, 1995), multi-attribute decision making (Diederich, 1997), and multi-alternative decision making (Roe et al., 2001), and decision rule learning (Johnson & Busemeyer, in press).

To introduce decision field theory, it will be helpful to consider an example problem. Suppose you have to choose a penalty program for a young offender, convicted of a serious crime, from one of three options: (A) a mild 5 year imprisonment, with a population of inmates that only have minor convictions, and a possibility for parole in 2 years; (B) a moderate 15 year imprisonment, with a population of inmates with moderately serious convictions, and a possibility for parole in 7 years; or (C) a severe 30 year imprisonment with a population of hardcore criminals with no possibility for parole. If we assume that the offender may be either corrigible (labeled event *g* for good) or incorrigible (labeled event *b* for bad), then Table 1 displays the six types of possible consequences for this decision. For example, if a mild penalty is chosen (option A) but the criminal is incorrigible (state *b*), then the outcome is the release of a dangerous man who will very likely repeat the crime.

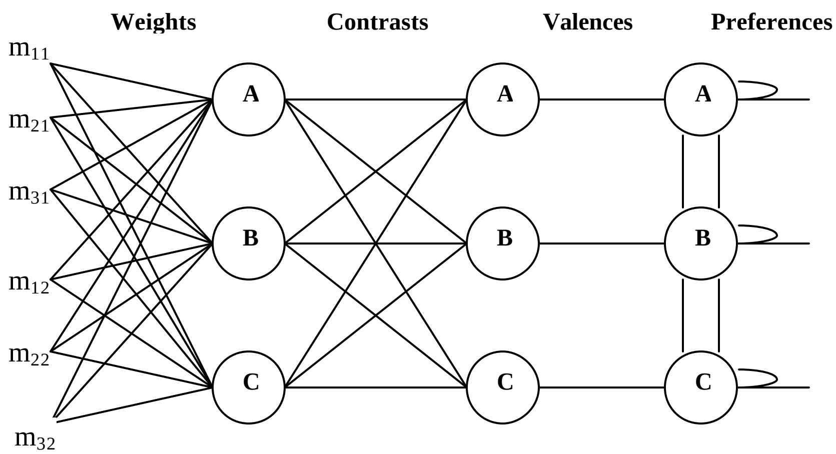
Table 1: Hypothetical Decision about Penalty for a Crime		
Action	Event *g*: Corrigible	Event *b*: Incorrigible
A: Mild Penalty	c_{11} : Reform to normal life	c_{12}: Release dangerous man
B: Moderate Penalty	c_{21}: Damage the man	c_{22}: Delay danger
C: Severe Penalty	c_{31}: Destroy a life	c_{32}: Safely incarcerate

According to decision field theory, the decision maker deliberates over these courses of action by thinking about the various possible consequences of each action. From moment to moment, different consequences come to mind over a period of time. For example, at one moment the decision maker may remember something (e.g., the kind face of the offender) that makes her think the offender can be reformed, and then she is appalled by the thought wasting his life, locked behind bars for 30 years. But at another moment, she may recall a recent story in which a parolee committed a horrible crime, and she may feel a cold fear arise from the idea of releasing another on the streets in a few years. At each moment, the affective reactions to the consequences of each action are evaluated and compared, and these comparisons are accumulated over time to form a preference state. The preference state for an action represents the integration of all the preceding affective reactions produced by thinking about that action during deliberation. This deliberation process continues until the accumulated preference for one action reaches a threshold, which determines the choice and the deliberation time of the decision (refer back to Figure 1).

The *threshold bound* for the decision process, symbolized θ, is a key parameter for controlling speed and accuracy tradeoffs. If θ is set to a low threshold, then only a weak preference is required to make a choice. In this case, decisions are made very quickly, which may be reasonable for trivial decisions of small consequence. However, a low threshold would cause the decision to be based on little thought about the consequences, which is likely to lead to a choice with bad unforeseen outcomes. For more serious decisions, θ is set to a very high threshold, so that a very strong preference is required to make a decision. In this case, deliberation takes longer, but the decision is based on a more thoughtful evaluation of all the consequences, producing a choice that is more likely to result in a positive outcome. Impulsive individuals may tend to use lower thresholds, while perspicacious individuals may tend to use higher thresholds.

The dynamical system used to generate this deliberation process is presented next, and the connectionist network is represented in Figure 2. The three actions corresponding to the mild, moderate, and severe penalty option are labeled A, B, and C, in this figure. The network has three layers of simple units that perform the following computations.

Figure 2: Connectionist Network Representation of Decision Field Theory



The inputs into this network, shown on the far left, represent the affective evaluations of the possible consequences of a decision. These *values* are assumed to be generated by a motivational system (hence the symbol m_{ij}), which is not explicitly represented here (but see Busemeyer, Townsend, & Stout, 2002). For example, m_{11} represents the positive evaluation of the consequence produced by reforming the offender and allowing him to return to society as a productive citizen, and m_{12} represents the negative evaluation of the consequence produced by releasing a dangerous man back into society.

The connections, linking the inputs to the first layer of nodes, are designed to represent an attention process. At any moment in time, the decision maker is assumed to attend to one of the possible events leading to consequences for each action. For example, if the decision maker thinks the criminal is incorrigible, then at that moment, option A is evaluated at m_{12}, option B is evaluated at m_{22}, and option C is evaluated at m_{32}. However, if something comes to mind which makes the decision maker switch her attention and think that the offender can be reformed, then at that later moment, option A is evaluated at m_{11}, option B is evaluated at m_{21}, and option C is evaluated at m_{31}. Thus, the inputs to the first layer fluctuate from one moment (time t) to another moment (time $t+h$) as the decision maker's attention switches from one possible event to another. The probability of attending to a particular event at each moment reflects the decision maker's underlying subjective probability or belief that the offender is corrigible. To formalize these ideas, we define $W_g(t)$ and $W_b(t) = 1 - W_g(t)$ as stochastic variables, called the *attention weights*, which fluctuate across time. For example, attention may be focused at time t on the corrigible event so that $W_g(t) > W_b(t)$, but a moment later at time $t+h$, attention may switch to the incorrigible event so that $W_b(t+h) > W_g(t+h)$. The first layer of the network computes a weighted value for each option i within a set of n options as follows

$$U_i(t) = W_g(t) \cdot m_{i1} + W_b(t) \cdot m_{i2} + e_i(t), \qquad (1)$$

The last 'error' term, $e_i(t)$, represents the influence of irrelevant features (e.g., in an experiment, these are features outside of an experimenter's control). The above equation looks like the classic weighted additive utility model, but unlike the classic model, the attention weights are stochastic rather than deterministic (see Fisher, Jia, & Luce, 2000, for a related model). The mean values of the attention weights correspond to the deterministic weights used in the classic weighted additive model.

The connections linking the first and second layers are designed to perform comparisons among weighted values of the options, to produce what are called valences. A positive valence for one option indicates that the option has an advantage under the current focus of attention, and a negative valence for another option indicates that the option has a disadvantage under the current focus of attention. For example, if attention is currently focused on event *g* (corrigible), then action A has an advantage over other options, and option C has a disadvantage under this state. But these valences reverse when attention is switched to event *b* (incorrigible). The second layer computes the *valence* for each option *i* within a set of *n* options by comparing the weighted value for option *i* with the average of the of the other (*n* -1) options:

$$v_i(t) = U_i(t) - U(t) \text{ ,} \qquad (2)$$

where $U(t) = \Sigma_{k \neq i} U_k(t) / (n\text{-}1)$. Valence is closely related to the concept of advantages and disadvantages used in Tversky's (1969) additive difference model. Note, however, that the additive difference model assumed complete processing of all features, whereas the present theory assumes a sequential sampling process that stops when a threshold is crossed.

The connections, between the second and third layers, and the interconnections among the nodes in the third layer, form a network that integrates the valences over time into a *preference state* for each action. This is a recursive network, with positive self-recurrence within each unit, and negative lateral inhibitory connections between units. Positive self-feedback is used to integrate the valences produced by an action over time, and lateral inhibition produces negative feedback from other actions. The third layer computes the preference state for option *i* from a set of *n* options according to the linear dynamic system:

$$P_i(t+h) = s \cdot P_i(t) + v_i(t+h) - \Sigma_{k \neq i} \; s_{ik} \cdot P_k(t) \text{ .} \qquad (3)$$

Conceptually, the new state of preference is a weighted combination of the previous state of preference and the new input valence. The initial preference state, $P_i(0)$, at the start of a decision problem, represents a preference recalled from past experience. This is used to explain carry over effects from previous decisions or past experience, such as the status quo effect (Samuelson & Zeckhauser, 1988).

Inhibition is also introduced from the competing alternatives. We assume that the strength of the lateral inhibition connection is a decreasing function of the dissimilarity between a pair of alternatives. For example, in Table 1, options A and C are more dissimilar than options A and B, and so the lateral inhibition between A and C would be smaller than that between options A and B. Lateral inhibition is commonly used in artificial neural networks and connectionist models of decision making to form a competitive system in which one option gradually emerges as a winner dominating over the other options (cf. Grossberg, 1988; Rumelhart & McClelland, 1986). As shown later in this chapter, this concept serves a crucial function for explaining several paradoxical phenomena of preferential choice.

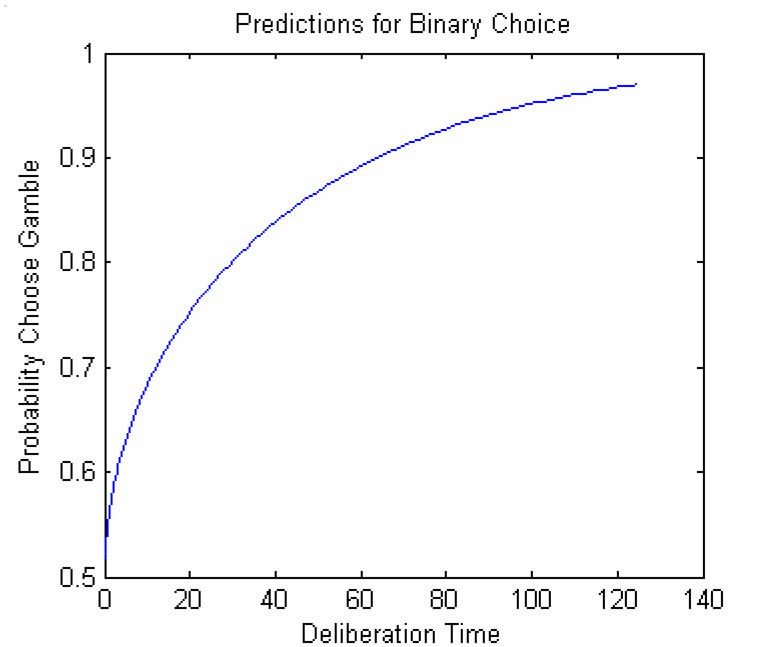
In summary, a decision is reached by the following deliberation process: as attention switches from one event to another over time, different affective values are probabilistically selected, and these values are compared across actions to produce valences, and finally these valences are integrated into preference states for each action. This process continues until the preference for one action exceeds a threshold criterion, at which point in time the winner is chosen. Formally, this is a Markov process, and matrix formulas have been mathematically derived for computing the choice probabilities and distribution of choice response times (for details, see Busemeyer & Townsend, 1992; Busemeyer & Diederich, 2002; Diederich & Busemeyer, 2003). Alternatively, Monte Carlo computer simulation can be used to generate predictions from the model.

(However, all of the predictions presented below were computed from the matrix formulas).

To illustrate the dynamic behavior of the model, consider a simple binary choice between a gamble and a sure thing. Suppose values for options A and B in Table 1 are set equal to the following: m_{11} = .96, m_{12}=0, m_{21}= .40, m_{22} = .40. With these values, option A can be viewed as a risky gamble, and option B can be viewed as a sure thing. Also assume an equal probability of attending to events *g* and *b*, i.e., $\Pr[W_g(t) = 1] = \Pr[W_b(t) = 1] = .50$. The variance of irrelevant dimensions (variance of ε) was set to zero, the self feedback was set to s = 1, and the lateral inhibition was set to $s_{AB} = s_{BA} = .01$.

Under these assumptions, we computed the choice probabilities and the mean deliberation times, for a wide range of threshold parameters (θ ranged from .20 to 8.0). Figure 3 plots the relation between choice probability and mean decision time for option A, the gamble, as a function of the threshold parameter. Both decision time and choice probability increase monotonically with the threshold magnitude. Busemeyer (1985) presents empirical evidence supporting these dynamic predictions for choices between a gamble and a sure thing under various time pressure conditions.

Figure 3: Predictions from decision field theory for binary choice

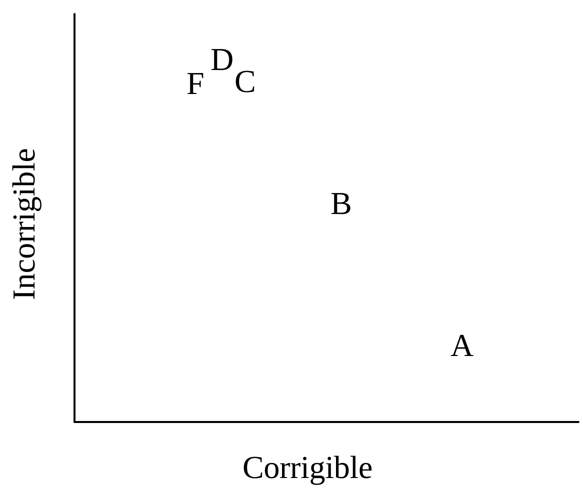


What do computational models contribute to decision theory?

Computational models are a lot more complex than the algebraic models commonly used by decision theorists. One could argue that computational models are too microscopic in their view, and they have little to show for their increased cost in complexity. Can computational models provide a gain in explanatory power that has not been achieved by the algebraic models? To answer this question, we will review a set of empirical phenomena that have resisted a coherent explanation by their algebraic counterparts.

To review these empirical phenomena within a common framework, it will be helpful to place the example decision problem, shown in Table 1, into a two dimensional representation, shown in Figure 4 below. The first dimension represents the evaluation of the options from the perspective that the offender is corrigible, and the second dimension represents the evaluation of the options from the perspective that the offender is incorrigible. Consider option A from Table 1: From the perspective that the offender is corrigible, then option A has a very high value; but from the perspective that the offender is incorrigible, then option A has a very low value. Thus option A is high on the first dimension and low on the second. Alternatively, option C has a low value from the corrigible perspective, but option C has a high value from the incorrigible perspective. Similarly, option B is midway between options A and C. We can also imagine other possible options in this space, which are variations of those shown in Table 1. Option D is another penalty program that is even more severe than option C; and option F is severe

Figure 4: Two dimensional Representations of Actions



like option C, but it is deficient, perhaps because there is less security at that institution. These examples will be used to illustrate the essential properties of the empirical phenomena reviewed below.

Similarity effect. This refers to the effect, on choice probabilities, produced by adding a competitive option D to an earlier choice set containing only A and C, where option D is very similar to option C. Suppose that in a binary choice between A and C, options A and C are chosen equally frequently so that Pr[C | {A,C}] = Pr[A | {A,C}]. Adding a new option D to this choice set, mainly takes away probability from the nearby option C, and leaves the probability of choosing option A unaffected. The empirical result is that the probability ordering for A and C changes from equality with the binary choice set, to Pr[A | {A,C,D}] > Pr[C | {A,C,D}] for the triadic choice set, producing a violation of a choice principle called independence of irrelevant alternatives (see Tversky, 1972, for a review). This robust empirical finding eliminates a large class of probabilistic choice models called simple scalability models, which includes for example, Luce's (1959) ratio of strength model. Tversky (1972) elegantly explained these results with a theory he called the elimination by aspects model of choice. Tversky (1972) also proved that the elimination by aspects model satisfies another important choice principle called regularity, which is considered next.

Attraction effect. This refers to the effect, on choice probabilities, of adding a decoy option F to an earlier choice set containing only options A and C, where the decoy F is similar to, but also dominated by, option C. Suppose, once again, that in a binary choice between A and C, options A and C are chosen equally frequently so that Pr[C|{A,C}] = .50. A second robust finding is that adding the decoy option F to this choice set enhances the probability of the nearby dominant option C, so that Pr[C|{A,C,F}] > Pr[C|{A, C}], which produces a violation of the regularity principle (Huber, Payne, & Puto, 1982; see Heath & Chatterjee, 1995, for a review). Consequently, this result cannot be explained by Tversky's (1972) elimination by aspects model. This violation of regularity also rules out a large class of random utility models of choice (Luce & Suppes, 1965), including Thurstone's (1959) preferential choice theory.

<u>Compromise effect.</u> This refers to the effect, on choice probabilities, of adding an intermediate option B to an earlier choice set containing only two extreme options A and C, where the compromise B is midway between the two extremes. Suppose, that all the binary choices are equal so that Pr[A | {A,B)] = Pr[A | {A,C)] = Pr[B | {B,C)] = .50. A third robust finding is that adding the compromise option B to a set containing A and C enhances the probability of the compromise option so that Pr[B | {A,B,C}] > Pr[A|{A,B,C}] = Pr[C | {A,B,C}], which is another violation of the independence between irrelevant alternatives principle (Simonson, 1989; see Tversky & Simonson, 1993 for a review). Tversky and Simonson (1993) proposed a context-dependent preference model based on the principle of loss aversion to explain the attraction and compromise effects. However, the context-dependent preference model cannot account for the similarity effect (see Roe et al., 2001, for a proof). Thus no model was proposed to account for all three simultaneously.

<u>A common explanation.</u> Decision field theory provides an explanation for all three phenomena using a common set of principles (see Roe et al., 2001, for details). In other words, we do not need to change any of the assumptions of the model across phenomena, and neither do we need to change any of the model parameters. The same assumptions always apply, and the same parameters can be used to predict all three effects. The mathematical basis for these predictions is derived elsewhere (see Roe et al., 2001; Busemeyer & Diederich, 2002), and here we only present an intuitive discussion.

First consider the similarity effect -- that is, the effect of adding option D to an earlier set containing A and C. The attention-switching property is essential for explaining this effect. On the one hand, whenever attention is focused on the corrigible event (corresponding to the first dimension in Figure 4), then option A alone gets a large positive advantage, while options C and D both have negative valences; on the other hand, whenever attention is focused on the incorrigible event (corresponding to the second dimension in Figure 4), then both options C and D have positive valences, while option A gets a large negative valence. If an individual happens to pay more attention to the corrigible event, then option A will tend to be chosen; but if an individual happens to pay more attention to the incorrigible event, then either option C or option D tend to be chosen. Therefore, option D only takes away probability from its neighboring option, C, and it does not affect the probability of choosing the more distant option, A.

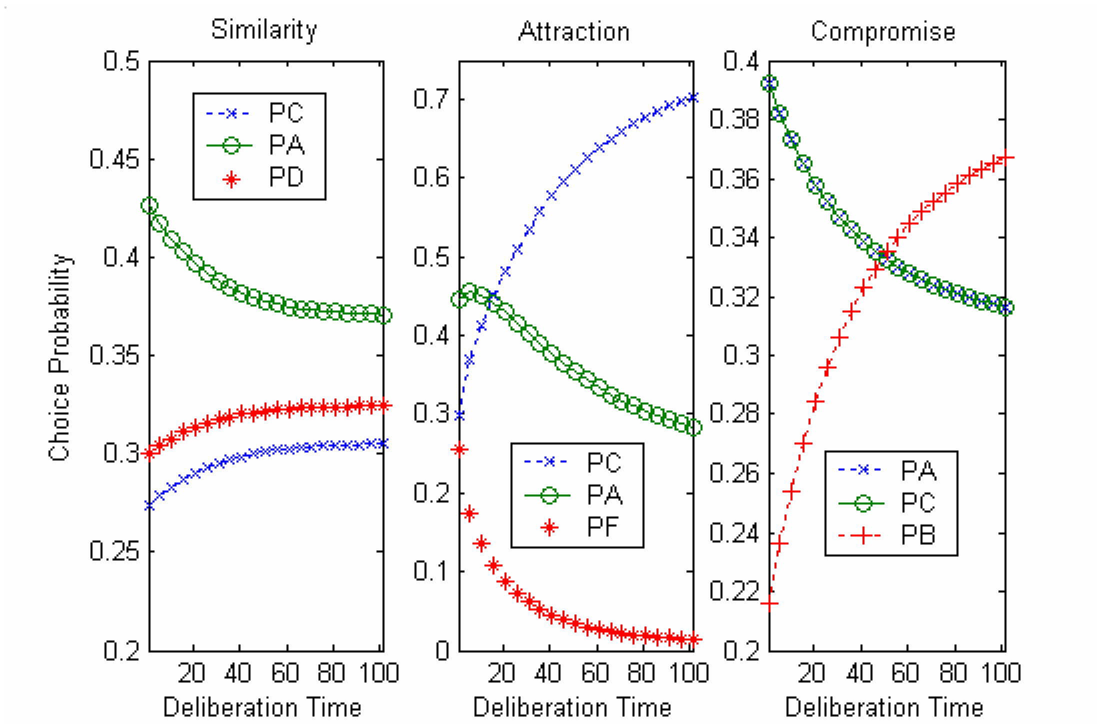
Next consider the attraction effect. In this case the lateral inhibition mechanism serves a crucial purpose. Neuroscientists long ago established the fact that the strength of lateral inhibitory connections decrease as a function of distance, and this property is responsible for generating contour and edge enhancement effects in vision (cf. Cornsweet, 1970). According to decision field theory, lateral inhibition produces an attraction effect for preference in the same way that it produces an edge enhancement effect for vision. During deliberation, the preference state for the dominated alternative F is driven toward a negative state because it competes with the nearby dominant alternative C. The negative preference state associated with option F feeds back through a negative inhibitory connection to option C, producing a bolstering (disinhibitory) effect on option C. This bolstering effect is not applied to option A because it is too distant from

F, and the lateral inhibitory link is too weak. Thus option C shines out by being close to an unattractive alternative, F.

Note that the attention switching and lateral inhibition processes are assumed to be operating all the time for both the similarity and attraction effects. These two components operate in synchrony to generate both effects. As a matter of fact, it is the interaction between these two processes that is essential for producing the compromise effect. In this case, if attention happens to focus on some irrelevant features favoring the compromise option, B, then this sends lateral inhibition to the neighboring extreme options A and C, decreasing their strength, which then builds up an advantage for the compromise option.

The predictions for all three effects were computed from decision field theory as follows. We simply set the values (m_{ij} in Equation 1) proportional to the coordinates shown in Figure 4, and the probabilities of attending to each dimension were equal ($\Pr[W_g(t) = 1] = \Pr[W_b(t) = 1] = .50$). The self feedback loop coefficient was set to s = .94, the lateral inhibitory coefficient for nearby options (e.g., s_{CD}) was set to .04, and the lateral inhibitory coefficient for distant options (e.g., s_{AC}) was set to .001. The standard deviation of the error, e, due to irrelevant dimensions was set equal to 1.25. Figure 5 shows the predictions for the triadic choice probabilities plotted as a function of deliberation time, separately for each effect. As can be seen in this figure, a common set of assumptions, and exactly the same parameters, reproduces all three effects.

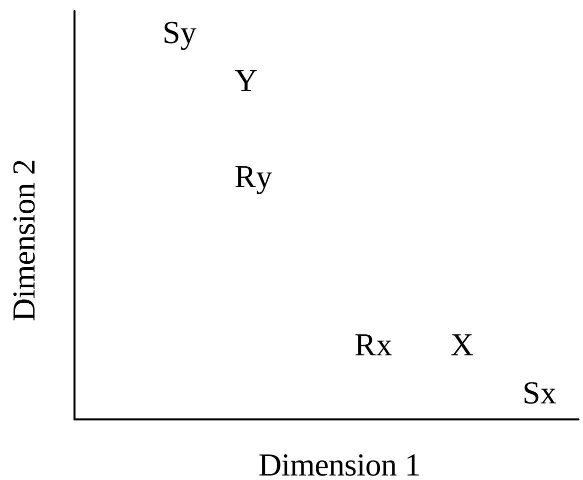
Figure 5: Predictions computed from decision field theory



An interesting prediction that follows from the above explanations for the attraction and compromise effects is that they should become stronger as deliberation time increases. In other words, if decision makers are encouraged to deliberate longer, then the attraction and compromise effects will increase. This is because lateral inhibitory effects grow in strength during deliberation. Two experiments have now been reported that confirm these dynamic predictions of the model (Simonson, 1989; Dhar, Nowlis, & Sherman, 2000).

Loss Aversion. An influential article by Tversky and Kahneman (1991) provides the most compelling evidence for loss aversion. The basic ideas are illustrated in Figure 6, where each letter shown in the figure represents a choice option described by two attributes; such as for example, consumer products that vary in size and quality, or jobs that vary in salary and interest. In this case, option X is high on dimension 1 but low on dimension 2, whereas option Y is low on dimension 1 but high on dimension 2.

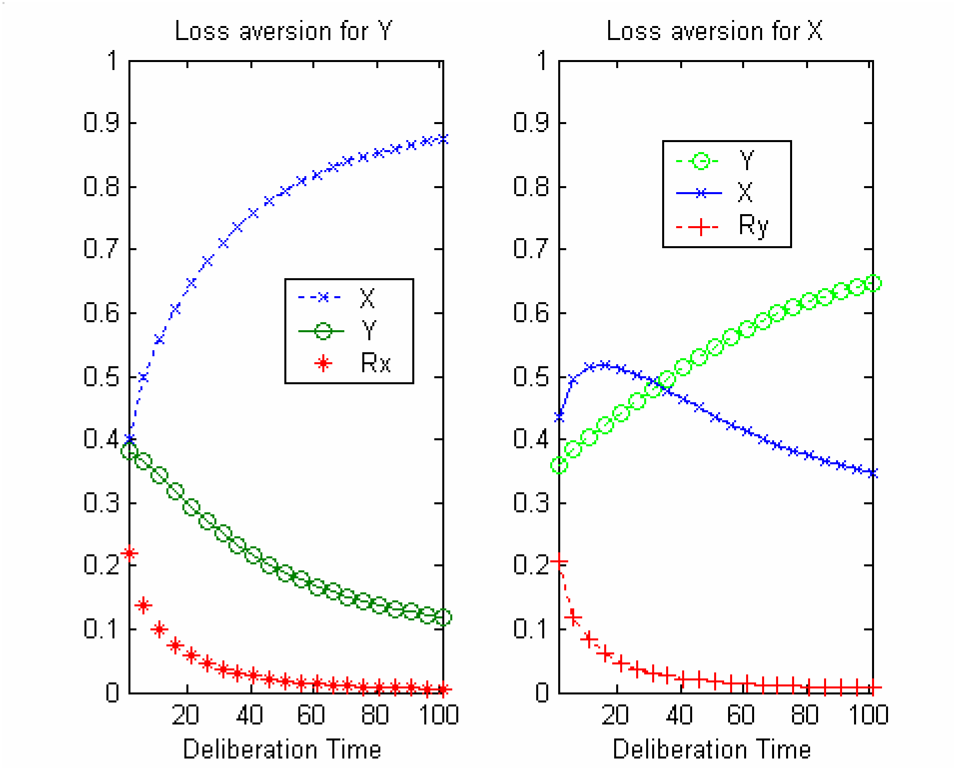
Figure 6: Options used to examine loss aversion



The first study manipulated a reference point, using either option R_x or R_y. Under one condition, participants were asked to imagine that they currently owned the commodity R_x, and they were then given a choice of keeping R_x or trading it for either commodity X or commodity Y. From the reference point of R_x, option X has a small advantage on dimension 1 and no disadvantage on dimension 2, whereas Y has both large advantages (dimension 2) and disadvantages (dimension 1). Under these conditions, R_x was rarely chosen, and X was strongly favored over Y. Under another condition, participants were asked to imagine that they owned option R_y, and they were then given a choice of keeping R_y or trading it for either X or Y. From the reference point of R_y, Y has a small advantage and no disadvantages, whereas X now has both large advantages and disadvantages. Under this condition, R_y was rarely chosen again, but now Y was slightly favored over X. (The smaller effect using R_y may indicate that dimension 2 was less important than dimension 1.) Tversky and Kahneman (1991) interpreted this pair of results as a loss aversion effect, because X was favored when Y entailed large losses relative to the reference point R_x, but the opposite occurred when X entailed large losses relative to the reference point R_y.

Decision field theory provides an explanation for this loss aversion effect through the lateral inhibition mechanism. To derive predictions from decision field theory, we simply set the values (m_{ij} in Equation 1) proportional to coordinates of the options in Figure 6. We set the probability of attending to the first dimension equal to .55, and the probability of attending to the second dimension equal to .45. The remaining parameters were the same as used to generate Figure 5. These predictions are shown in Figure 7, which shows the probability of the triadic choices as a function of deliberation time, separately for the two reference point conditions. As can be seen in this figure,

Figure 7: Decision field theory predictions for loss aversion effect.



decision field theory reproduces the loss aversion effect — that is, the change in preference for option X relative to Y depending on the reference point. It is important to note that exactly the same parameters are used for both reference point conditions. This reversal of preference does not depend on the probability of attending to each dimension —if we set the probabilities equal to .50 then the reversal becomes even stronger, although symmetric in size. In fact, the result depends primarily on the lateral inhibition parameter — if it is set to zero, then the effect disappears.

The second study also manipulated a reference point, but in this case, using either option S_x or S_y. In one condition, participants were asked to imagine that they trained on job S_x, but that job would end, and they had to choose between two new jobs X or Y. From this reference point, job X has small advantages and disadvantages over S_x, whereas Y has large advantages and disadvantages. Under these conditions, option X was strongly favored over option Y. In a second condition, participants were asked to imagine that they trained on job S_y, and in this case, preferences reversed, and option Y was strongly favored over option X. Tversky and Kahneman (1991) also interpreted these results as a loss aversion effect.

To apply decision field theory to this study, we assume that each option is described by three dimensions: the values of the first two dimensions (e.g., salary and interest) are taken from the positions of the options shown in Figure 6, and the third dimension represents job availability. Jobs X and Y both have a positive value on dimension 3 (they are available), whereas jobs S_x and S_y both have negative values on dimension 3 (they are no longer available). For example, option S_x is assigned a slightly higher value on dimension 1 than option X, a slightly lower value on dimension 2 than option X, and it has a large negative value on dimension 3. We assumed an equal probability of attending to each of the three dimensions, and the remaining parameters were the same as used to generate Figure 5. The asymptotic choice probability results, predicted the theory, are summarized in Table 2, below.

Table 2: Predictions Computed from Decision Field Theory		
	S_x Reference Point	S_y Reference Point
Option	Choice Probability	Choice Probability
X	.87	.13
Y	.13	.87
S	0	0

As can be seen in the table, decision field theory again reproduces the reversal in preference as a function of the reference point. In sum, we find that both loss aversion effects, as well as attraction and compromise effects, all can be derived from the lateral inhibitory mechanism of decision field theory.

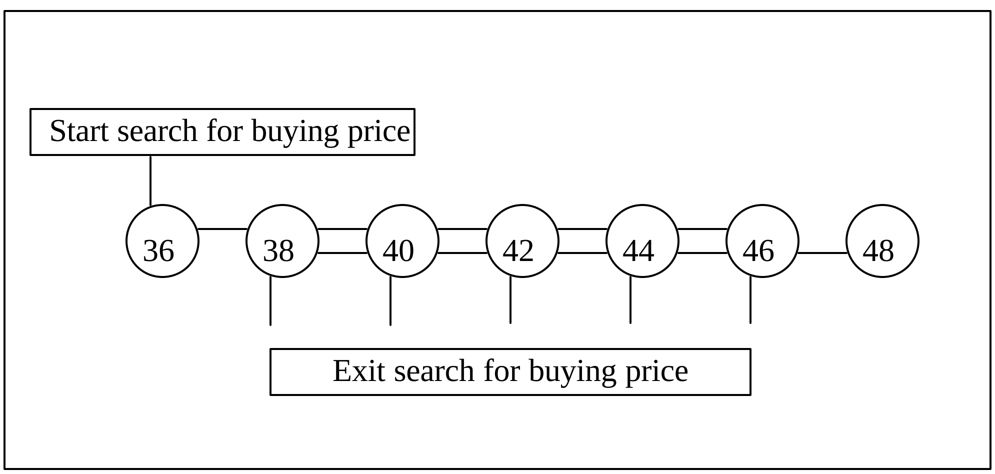
Endowment effect. There are other phenomena that are often interpreted in terms of loss aversion (cf. Tversky & Kahneman, 1991), including both the endowment effect as well as differences between willingness to buy versus willingness to pay. Kahneman, Knetsch, and Thaler (1990) gave one group of subjects a mug and asked them how much they would be willing to pay to give up the mug, whereas another group was simply given some money and asked how much they would be willing to pay to buy the mug. They found that subjects were willing to buy the mug for only about $3, but they were asking a much higher price of $7 to sell the mug. This price difference is interpreted as the loss aversion effect produced by an owner giving up his or her mug. As Tverksy & Kahneman (1991) noted, the endowment effect can be viewed a special case of a more general finding of disparities between the price individuals are willing to accept to sell something they own (WTA or selling prices), versus the price they are willing to pay to acquire something do not own (WTP or buying prices).

At first glance, one might argue that differences between buying and selling prices are simply a strategic effect: a person may deliberately underestimate the buying price and overestimate the selling price to gain an advantage. But this simple explanation implies that buying and selling prices would still produce the same rank orders. In fact, this is not the case. Birnbaum, Yeary, Luce, & Zhou (2002) review several studies that report preference reversals between buying versus selling prices. For example, Birnbaum and Sutton (1992) presented subjects with the following two gambles: gamble G gives a .5 probability of winning $96, otherwise $0; gamble F gives a .5 probability of winning $48, otherwise $36 dollars. On the average, subjects gave a higher buying price to gamble F than gamble G, but at the same time they gave a higher selling price to gamble G than gamble F. Birnbaum and Sutton (1992) explained these effects as a change in decision weight that depends on the buyer or seller point of view.

This type of preference reversal is predicted by decision field theory even when the inputs to the process used to produce buying and selling prices are based on a common set of weights and values. The reversals emerge from the dynamic process used to select the prices. A brief presentation of the computational model used in decision field theory to select prices for gambles is presented below (see Busemeyer & Goldstein, 1992; and Townsend & Busemeyer, 1995, for more details).

The basic idea is that prices are selected by a series of covert comparisons (refer to Figure 8). To find a price equivalent to a gamble, the decision maker must search for a candidate that produces an indifference response. During each step of this search process, the decision maker compares a candidate price to the gamble, and this comparison may result in one of three judgments: if the candidate price is preferred, then the price is decremented by a small amount and the search continues (a left transition in Figure 8); if the gamble is preferred, then the price is incremented by a small amount and the search continues (a right transition in Figure 8); if the comparison produces an indifference judgment, then the search stops and the candidate price is reported as the price (a downward transition in Figure 8). We simply use decision field theory to perform this comparison process, which provides the probabilities for the three judgments at each stage of the search process (see Busemeyer & Goldstein, 1992, for details). Then Markov chain theory is used to determine the distribution of prices generated by the search process (see Busemeyer & Townsend, 1992, for the mathematical derivations).

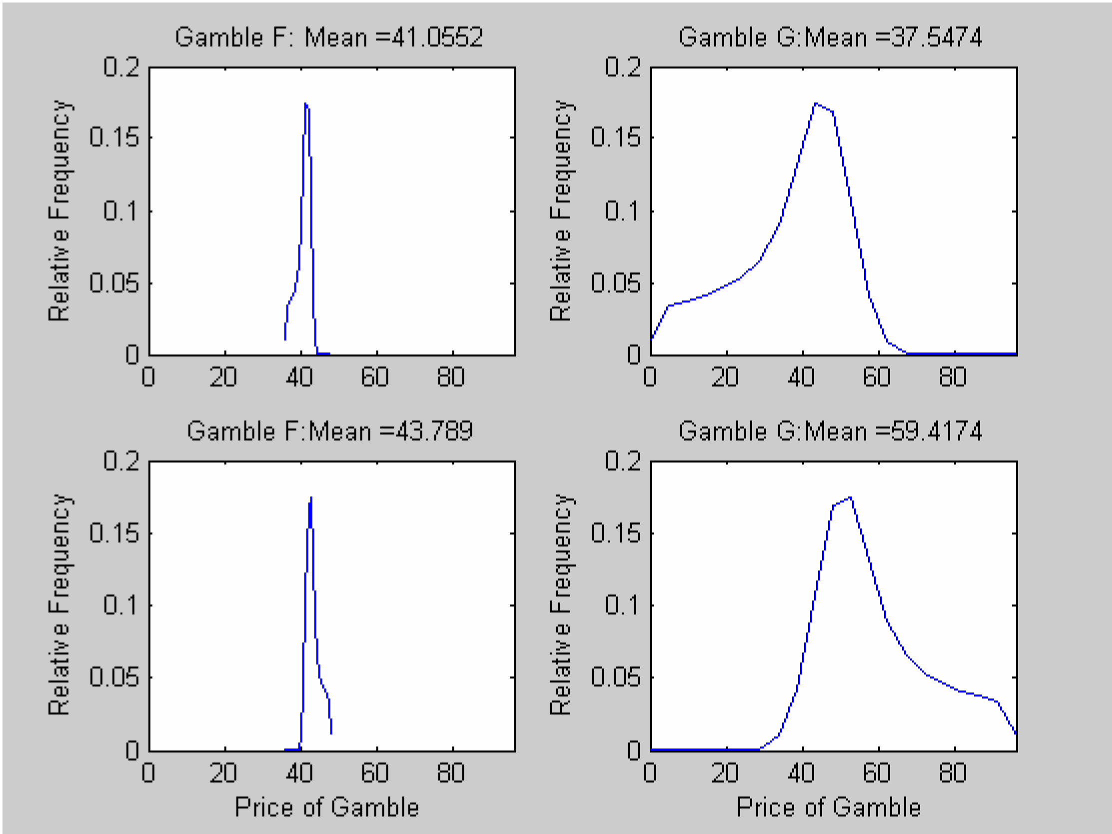
Figure 8: Illustration of the search process for finding the price of a gamble.



When asked to find a certainty equivalent for a gamble, we assume that the search process starts near the middle of the feasible set of prices in an attempt to minimize the number of steps needed to find the price equivalent. When asked to find a maximum buying price for a gamble, we assume that the search process starts near the minimum of the feasible set of prices, biased away from paying excess money. Finally, when asked to find a minimum selling price, we assume that the search process starts near the maximum of the feasible set of prices, biased toward saving extra money.

This simple scheme was used to find buying and selling prices for gambles F and G used by Birnbaum and Sutton (1992). In this case, we simply set the values (m_{ij} in Equation 1) equal to the stated dollar values of the gambles, and we simply set the probability of attending to each event equal to the stated probabilities. Figure 9 shows the distribution of prices produced by this model for buying prices (top panel) and selling prices (bottom panel).

Figure 9: Predicted Buying Prices (top panels) and Selling Prices (bottom panels)



As can be seen in Figure 9, the predicted buying prices (or WTP) are lower than the predicted selling prices (or WTA), accounting for the well known disparity between these measures. More importantly, preference reversals occur for buying and selling prices: referring to the top panels, the mean buying price for gamble F is larger than the buying price for gamble G; referring to the bottom panels, the mean selling price for gamble G is greater than the mean selling price for gamble F.

There is an intuitive explanation for these computational results. The price for gamble F is restricted to a small range, which makes the price insensitive to changes in the starting position produced by the selling or buying price task. However, the price for gamble G has a wide range of possible values, and it is more strongly affected by the starting position produced by buying and selling tasks. This idea is similar to earlier anchoring and adjustment models of preference reversal (e.g., Goldstein & Einhorn, 1987). However, unlike these earlier anchoring and adjustment theories, the amount of adjustment is not a free parameter in decision field theory, because it is derived from the dynamics of the search process.

Preference reversals also occur between prices and choices (Lichtenstein and Slovic, 1971; see Slovic and Lichtenstein, 1983, for a review). Decision field theory can also reproduce these types of preference reversals by using a common set of weights and values as inputs into the choice and price processes (Busemeyer & Goldstein, 1992). Decision field theory can also explain discrepancies reported by Hershey and Shoemaker (1985) between certainty equivalents and probability equivalents for gambles (Townsend & Busemeyer, 1995).

Preference reversals under time pressure. Up to this point we have argued that computational models, such as decision field theory, provide a deeper level analysis of several traditional effects from the decision-making literature. Now we turn to new predictions that arise from the dynamic nature of the model.

There is a growing body of evidence showing that it is possible to reverse an individual's preference by changing the amount of time given to make the decision. For example, Svenson and Edland (1987) asked people to choose among apartments under short vs. long time deadlines. Under the short time deadlines, the lower rent apartment was chosen more frequently; but under longer time deadlines, they preferred apartments with higher rents that provided other attractive features. Diederich (2003) extended these findings by asking individuals to choose between two gambles, and each gamble could yield either a monetary reward or a blast of noise punishment. Several individuals reversed their preferences under time pressure. For example, if avoiding noise was more important than winning money, then the low noise gamble was chosen more frequently under short deadlines, but the high monetary payoff gamble was chosen more frequently under the longer deadlines.

A common explanation for these effects is that decision makers switch strategies (Payne, Bettman, & Johnson, 1993). Under short deadlines, it is hypothesized that decision makers use a non-compensatory heuristic strategy such as a lexicographic rule or an elimination by aspects rule. These strategies are quick and easy to execute but are not very accurate in the sense of maximizing weighted additive utility. Under longer deadlines, decision makers can use the more time consuming compensatory strategy such as a weighted additive rule which increases accuracy.

Sequential sampling models provide an alternative view, which simply assumes that individuals reduce their threshold criterion under time pressure. Diederich (1997) developed a multi –attribute version of decision field theory, which assumes individuals sequentially sample information over time, but they begin processing the more important dimension, and later switch to process the other less important dimensions. Under short deadlines, a low threshold is used, only the most important dimension tends to get processed, and so this dimension alone determines the choice. Under long deadlines, a high threshold is used, and now there is sufficient time to process additional attributes. If these additional attributes disagree with the most important attribute, then this additional processing can reverse the direction of the evolving preference state. Diederich (1997) showed that this model provided a very accurate quantitative account of her preference reversals under time pressure.

Are computational models testable?

One might argue that computational models are so complex that they cannot be empirically tested. On the contrary, it is possible to rigorously test these models both quantitatively as well as qualitatively. For example, to quantitatively test decision field theory, one can estimate all of the model parameters from a set of binary choice probabilities, and then use these same parameters to predict other measures of preference including choice response times, triadic choice probabilities, and buying/selling prices (see, for examples, Dror, Busemeyer, & Basola, 1999; Diederich & Busemeyer, 1999; Diederich, 2003a; and Diederich, 2003b). Qualitative tests of the theory are also possible: on the one hand, decision field theory predicts violations of strong stochastic transitivity, but on the other hand it predicts that weak stochastic transitivity will be satisfied (Busemeyer & Townsend, 1993). In agreement with the first qualitative prediction, violations of strong stochastic transitivity frequently occur (see Mellers & Biagini, 1994, for a review); but contrary to the second qualitative prediction, violations of weak stochastic transitivity also have been reported under special conditions (see Gonzalez – Vallejo, 2002, for a recent review and explanation for this result).

What are some alternative computational models?

Up to this point we have highlighted one computational model, decision field theory, but there are a growing number of new computational models for decision making. Three of these are briefly described below.

Competing accumulator model. Usher and McClelland (2001, 2001) have recently proposed a competing accumulator model that shares many assumptions with decision field theory, but departs from this theory on a few crucial points. The connectionist network of the competing accumulator model is virtually the same as shown in Figure 2. However, this model makes different assumptions about (a) the evaluations of advantages and disadvantages (what we call valences in Equation 2), and (b) the dynamics of response activations (what we call preference states in Equation 3). First, they adopt Tversky and Kahneman's (1991) loss aversion hypothesis so that disadvantages have a larger impact than advantages. Using our own notation, the valence for alternative $i \in \{A,B,C\}$, and $i \neq j \neq k$, is computed as follows:

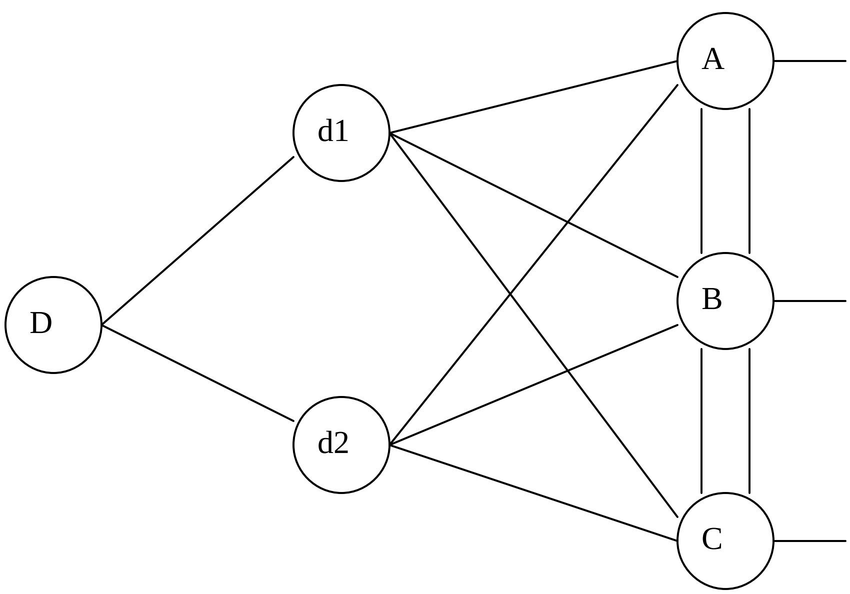
$$v_i(t) = F[U_i(t) - U_j(t)] + F[U_i(t) - U_k(t)] + c \qquad (4)$$

Where $F(x)$ is a nonlinear function that satisfies the loss aversion properties presented in Tversky & Kahneman (1991). Thus, rather than deriving loss aversion effects indirectly from the dynamics as we have done, they build this effect directly into the model. Second, they use a nonlinear dynamic system that restricts the response activation to remain positive at all times, whereas we use a linear dynamical system that permits positive and negative preference states. The non-negativity restriction was imposed to be consistent with their interpretation of response activations as neural firing rates.

Usher and McClelland (2002) have shown that the competing accumulator model can account for the main findings concerning the similarity effect, the attraction effect, and the compromise effect, using a common set of parameters. Like decision field theory, this model uses an attention switching mechanism to produce similarity effects, but unlike decision field theory, this model uses loss aversion to produce the attraction and compromise effects. Further research is needed to discriminate between these two models.

ECHO model. Guo and Holyoak (2002; see also Glockner, 2002) proposed a different kind of connectionist network, called ECHO, adapted from Thagard and Millgram (1995). Figure 10 illustrates the model for two attributes and three options. At the far left in this figure, there is a special node, called the external driver, representing the goal to make a decision, which is turned on when a decision is presented. The driver node is directly connected to the attribute nodes, with a constant connection weight. Each attribute node is connected to an alternative node with a bidirectional link, which allows activation to pass back and forth from the attribute node to the alternative node.

Figure 10: Illustration of the Echo Model for 2 dimensions and 3 alternatives



The connection weight between an attribute node and an alternative node is determined by the value of the alternative on that attribute (our m_{ij}). There are also constant lateral inhibitory connections between the alternative nodes.

The decision process works as follows. Upon presentation of a decision problem, the driver is turned on and applies constant input activation into the attribute nodes, and each attribute node then activates each alternative node (differentially depending on value). Then each alternative node provides positive feedback to each attribute node, and negative feedback to the other alternative nodes. Activation in the network evolves over time according to a nonlinear dynamic system, which keeps the activations bounded between zero and one. The decision process stops as soon as the changes in activations fall below some threshold. At that point, the probability of choosing an option is determined by a ratio of activation strengths.

Guo and Holyoak (2002) used this model to explain the similarity and attraction effects. To account for these effects, they assumed that the system first processes the two similar alternatives, and during this time, the lateral inhibition produces a competition between these two options. After this initial comparison process is completed, the system processes all three options, including the dissimilar option. In the case of the similarity effect, the initial processing lowers the activation levels of the two similar options; in the case of the attraction effect, the initial processing enhances the activation level of the dominating option. Thus lateral inhibition between alternatives plays a crucial role for explaining both effects. Although the model has been shown to account for the similarity and attraction effects, at this point, it has not been shown to account for the compromise effect or loss aversion effects.

The ECHO model makes an important prediction that differs from both decision field theory and the competing accumulator model. The ECHO model predicts that as one option becomes dominant during deliberation, this will enhance the activation of the attribute nodes favored by the dominant alternative. The enhancement is caused by the feedback from the alternative node to the attribute node, which tends to bias the evaluation of the attributes over time. This property of the model is related to the dominance-seeking principle included in other decision-making theories (Montgomery, 1989; Svenson, 1992). Holyoak and Simon (1999) tested this hypothesis by asking individuals to rate attribute importance at various points during deliberation, and they report evidence for increases in the importance of attributes that are favored by the dominant alternative during deliberation.

Affective Balance Theory. Grossberg and Gutowski (1987) presented a dynamic theory of affective evaluation based on an opponent processing network called a gated dipole neural circuit. Habituating transmitters within the circuit determine an affective adaptation level, or reference point, against which later events are evaluated. Neutral events can become affectively charged either through direct activation or antagonistic rebound within the habituated dipole circuit. This neural circuit was used to provide an explanation for the probability weighting and value functions of Kahneman and Tversky's (1979) prospect theory, and the affective dynamics of addiction and withdrawal symptoms hypothesized by Solomon and Corbit (1974).

Computational models of inference. Although this chapter focused on computational models of preference, there are also new developments for probabilistic inference and prediction. Dougherty, Gettys, and Ogden (1999) developed an instancebased memory model for probability judgments that accounts for overconfidence effects and conjunctive fallacies. Read, Vanman, and Miller (1997) developed a connectionist model for social inference judgments which is closely related to the ECHO model used by Holyoak and Simon (1999). Busemeyer, Byun, Delosh, and McDaniel (1997) proposed a connectionist model for cue - criterion prediction tasks.

Concluding Comments

During the past 40 years, decision theorists have let the utility function do most of the work of explaining choice results. By positing the simplest possible hypotheses about the choice processes, all the explanatory power falls upon the utility function itself. Consequently, during this 40-year span of time, the forms of utility functions have become increasingly complex (see Luce, 2000, for a review). However, it is possible that if theorists work harder in understanding the complexities inherent in the choice processes, then the underlying utility representations may become simpler and more coherent. As others have argued (cf. Plott, 1996), it may be too early for decision theorists to accept the conclusion that utilities are constructed on the fly for every variation of task and context, and instead it may be possible to retain a stable underlying value system that is expressed through a very complex choice process.

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