Research Report

CONFLICT AND THE STOCHASTIC-DOMINANCE PRINCIPLE OF DECISION MAKING

Adele Diederich[1] and Jerome R. Busemeyer[2]

[1]Univesitat Oldenburg, Oldenburg, Germany, and [2]Indiana University

Abstract—*One of the key principles underlying rational models of decision making is the idea that the decision maker should never choose an action that is stochastically dominated by another action. In the study reported in this article, violations of stochastic dominance frequently occurred when the payoffs produced by two actions were negatively correlated (in conflict), but no violations occurred when the payoffs were positively correlated (no conflict). This finding is contrary to models which assume that choice probability depends on the utility of each action, and the utility for an action depends solely on its own payoffs and probabilities. This article also reports, for the first time ever, the distribution of response times observed in a risky decision task. Both the violations of stochastic dominance and the response time distributions are explained in terms of a dynamic theory of decision making called multiattribute decision field theory.*

Traditionally, decision theories have been divided into two categories: rational theories— founded on sound principles of reasoning—and descriptive theories—designed to describe human behavior. Rational theories have been promoted by economists, whereas descriptive theories are favored by psychologists. Although the two theoretical approaches have different goals, each has a tremendous influence on the other. On the one hand, paradoxical decision behavior has led to a decline in the once-dominant rational theory—expected utility theory— and the rise of a new rational theory—rank-dependent utility theory (see Luce, 1992). On the other hand, a prominent descriptive theory of decision making—prospect theory—was cast down in favor of another descriptive theory—cumulative prospect theory—because the former violated a rational principle known as *stochastic dominance* (Tversky & Kahneman, 1992). The purpose of this article is to test the empirical status of the stochastic-dominance principle.

STOCHASTIC DOMINANCE

Suppose there are N possible consequences that can be produced by either of two actions A or B, and let $x_1 <_p x_2 <_p \ldots <_p x_i <_p \ldots <_p x_N$ represent a preference-ordered list of the N consequences, where x_1 is the worst and x_N is the best. The probability of obtaining a consequence as good as x_i or better from Action A is symbolized by $G_A(x_i)$, and $G_B(x_i)$ is the corresponding probability for Action B. Then Action A stochastically dominates Action B if and only if $G_A(x_i) \geq G_B(x_i)$ for all x_i, and the inequality is strict for at least one x_i (see Clemen, 1996, p. 124; Wakker, 1989).

Table 1 illustrates the idea in a simple form. The first situation in the table is an example of a medical decision in which the decision maker must choose between two treatments (A vs. B) for an injury. The outcome of each treatment is described in terms of two attributes, treatment cost and recovery duration. The outcome produced by each treatment depends on which of two equally likely internal states (X or Y) exists for the injured person.

In this example, it seems safe to assume that x_1 = (190 days of recovery, \$255,000 cost) $<_p x_2$ = (180 days of recovery, \$250,000 cost) $<_p x_3$ = (14 days of recovery, \$5,500 cost) $<_p x_4$ = (7 days of recovery, \$5,000 cost). In this case, for example, $G_A(x_3)$ is interpreted as the probability of getting a payoff at least as good as x_3 from Action A (this is equivalent to the probability of getting x_4 from Action A, because x_4 is the only outcome from Action A that is at least as good as x_3). Given equally likely states,

$$G_A(x_1) = 1.0 = G_B(x_1) = 1.0,\ G_A(x_2) = 1.0 > G_B(x_2) = .50,\ G_A(x_3) = .50 = G_B(x_3) = .50,\ G_A(x_4) = .50 > G_B(x_4) = 0.0 \quad (1)$$

Thus, Action A stochastically dominates Action B.

Strictly speaking, a perfectly rational decision maker should never choose an action that is stochastically dominated by another action. However, random errors can occur in the evaluation of each action, and this could lead to occasional violations of the stochastic-dominance principle. To deal with this issue (see, e.g., Birnbaum & Thompson, 1996), previous empirical choice tests of stochastic dominance were based on a probabilistic choice model called the strong utility model (Luce & Suppes, 1965). According to this model, the probability of choosing Action A over Action B, denoted Pr[A,B], can be expressed as an increasing function of the subjective utility of Action A, and a decreasing function of the subjective utility of Action B:

$$Pr[A,B] = F[u(A), u(B)], \quad (2)$$

where u(A) is a function of the probabilities and consequences produced by Action A (e.g., the first row of Table 1), u(B) is a function of the probabilities and consequences produced by Action B (e.g., the second row of Table 1), and F is an increasing function of the first variable and a decreasing function of the second variable. For example, u(A) and u(B) in Equation 2 may be defined by Tversky and Kahneman's (1992) cumulative prospect model. Utility models, which satisfy the principle of stochastic dominance (e.g., cumulative prospect theory), assume that greater utility is assigned to the stochastically dominant alternative.

The bottom half of Table 1 provides another example in which Action A stochastically dominates Action B; in fact, it yields exactly the same order as shown in Equation 1. Furthermore, the strong utility model of risky decision making (i.e., Equation 2) predicts that the probability of choosing Action A over Action B should be identical for the two situations in Table 1. This is because the probabilities and consequences for Action A are identical for the two examples, and so are the probabilities and consequences for Action B. Nevertheless, these two decisions seem psychologically different: An individual experiences more conflict when faced with the first situation compared with the second.

Address correspondence to Jerome R. Busemeyer, Psychology Department, Indiana University, Bloomington, IN 47405-1301; e-mail: jbusemey@indiana.edu.

Table 1. *Hypothetical example illustrating the effect of payoff correlation on preference*

	State	
Treatment	X (*Pr* = .50)	Y (*Pr* = .50)
	Negatively correlated actions	
A	7 days of recovery, $5,000 cost	180 days of recovery, $250,000 cost
B	190 days of recovery, $255,000 cost	14 days of recovery, $5,500 cost
	Positively correlated actions	
A	7 days of recovery, $5,000 cost	180 days of recovery, $250,000 cost
B	14 days of recovery, $5,500 cost	190 days of recovery, $255,000 cost

MULTIATTRIBUTE DECISION FIELD THEORY

According to decision field theory (DFT; see Busemeyer & Townsend, 1993; Townsend & Busemeyer, 1995), and its recent generalization called multiattribute decision field theory (MDFT; see Diederich, 1997; Diederich & Busemeyer, 1999), violations of stochastic dominance are predicted to occur when conflict is high (as in the top half of Table 1), but no violations are expected when conflict is absent (as in the bottom half of Table 1).

According to MDFT, decisions are based on a preference state, P, representing the relative preference for one action (e.g., Action A) over the other (e.g., Action B) in a binary choice. The preference state evolves across time (denoted t) according to the following linear dynamic model (h is a small time unit):

$$P(t + h) = (1 - s \cdot h)P(t) + V(t + h), \tag{3}$$

where $V(t)$ is the input valence representing momentary comparison of the consequences produced by each action. The parameter s in Equation 3 determines the rate of growth or decay in the preference over time. A final decision is reached as soon as the preference state exceeds a threshold, $\theta > 0$: If $P(t) > \theta$, then choose Action A, if $-P(t) > \theta$, then choose Action B.

The valence $V(t)$ fluctuates because of moment-to-moment changes in the decision maker's attention, which switches back and forth from one state (e.g., State X) to another (e.g., State Y), and from one attribute (e.g., treatment cost) to another (e.g., recovery duration), during deliberation. While attending momentarily to a particular state and attribute, the individual compares consequences produced by each action under that state and attribute, producing the valence $V(t)$ at that moment.

In the high-conflict case of the top half of Table 1, the valence $V(t)$ changes sign back and forth from positive to negative as attention switches from State X, favoring Action A, to State Y, favoring action B. In the no-conflict case of the bottom half of Table 1, the comparison always produces a positive (or zero) increment favoring Action A over Action B, so $V(t)$ is always nonnegative.

It follows from MDFT that the decision in the former situation is much more difficult than the decision in the latter situation, because the former evokes more conflict in the following sense. In this case, if State X obtains, then Action A produces a relatively good outcome and Action B produces a very bad outcome; but the opposite happens if State Y obtains. If the decision maker momentarily contemplates the possibility that State Y can occur, then Action B seems more desirable. These momentary comparisons sometimes favor Action A, and sometimes favor Action B, producing up-and-down vacillation in the preference state over time. Therefore, given the values in the top half of Table 1, MDFT predicts that stochastic dominance will be violated frequently (violations occur on those trials when the decision maker happens to attend to State Y more than State X).

In the case of the bottom half of Table 1, if the decision maker momentarily attends to the possibility of State X occurring, then Action A is expected to produce a better consequence than Action B; if the decision maker momentarily attends to the possibility of State Y occurring, the same is true. Thus, Action A always produces a better outcome, independent of the state to which the decision maker attends. The comparison always favors Action A over Action B, so the preference state will always increase in the direction favoring Action A over Action B. Therefore, given the values in the bottom half of Table 1, MDFT predicts that stochastic dominance will be perfectly satisfied.

The critical factor being manipulated across the two halves of Table 1 is the correlation between the payoffs produced by the two actions. In the top half of the table, the outcomes are negatively correlated: Whenever a good outcome occurs for one action, a bad outcome occurs for the other, producing high conflict. In the bottom half of the table, the outcomes are positively correlated: When State X occurs, both actions produce good outcomes, but when State Y occurs, both actions produce bad outcomes, producing low conflict.

The primary purpose of this experiment was to test the predictions of MDFT concerning the effect of manipulating the correlation between the payoffs on frequency of stochastic-dominance violations. The stochastic-dominance principle and the strong utility model of risky decision making predict that this correlation manipulation should have no effect. According to MDFT, frequent violations are expected under the negatively correlated (high-conflict) condition, but no violations are expected under the positively correlated (no-conflict) condition.

METHOD

A secondary purpose of this experiment was to examine the distribution of choice response times under the positively versus negatively correlated conditions. The process used to decide under negatively correlated (conflict-present) conditions may be qualitatively different from the process used to decide under positively correlated (conflict-absent) conditions. Choice time distributions may reveal systematic changes in the dynamics of the decision process depending on the presence or absence of conflict. To our knowledge, this is the first experiment to examine choice time distributions in a risky decision task.

Therefore, the present study consisted of two parts. The first part examined the generality of stochastic-dominance violations across a wide range of payoff conditions with 2 paid subjects. For this purpose, it was necessary to employ a methodology more commonly used in psychophysics—presenting a very large number of choice trials to a small number of participants (see, e.g., Ratcliff & Rouder, 1998). The second part of the study examined the generality of these results with a large number (N = 17) of subjects under a single payoff condition.

Subjects

Subjects for the first part of the study were a male (J.B.) and a female (H.G.), both students at Univesitat Oldenburg. Each student participated in five conditions. There were six sessions per condition and 120 choice trials per session. Subjects received 10 DM (Deutsche mark) per session (about $8), plus the amount of money they earned during each session.

Subjects for the second part of the study were 17 students from the Univesitat Oldenburg. Each subject participated in only one condition entailing six sessions. Subjects were given class credit for participating in the experiment.

Procedure

Each trial began with a simple display of two letters on a computer screen, and the participant was required to choose one of the two letters by pressing a button on a response box. Two seconds later, feedback was provided; the screen displayed the numerical values of the payoffs produced by each action on that trial. This feedback was followed by the actual delivery of the payoff produced by the letter chosen on that trial. The probabilities of the payoffs produced by each action were learned through experience from the trial-by-trial feedback. The choice and the response time (in milliseconds) for each trial were recorded by the computer.

The payoff had two attributes: a gain or loss of money (DM units) and a duration of auditory noise (67 dB). The noises were presented binaurally by closed headphones; they were generated by a synthesizer and digitally sampled by a sound card. (For the second part of the study, the points that subjects earned represented only fictitious money, but subjects still experienced an actual burst of noise.)

The 2 subjects in the first part of the study received five conditions presented in different orders. The subjects in the second part received only the fifth condition. Each condition consisted of 240 training trials plus 480 experimental trials. During each condition, one of two different choice pairs was presented on each trial in random order. For convenience, the two choice pairs are denoted here as (A_j, B_j) and (A_j, C_j) for condition $j = 1, \ldots, 5$. For each condition, A_j was designed to stochastically dominate both B_j and C_j, but Action B_j was negatively correlated with Action A_j, and Action C_j was positively correlated with Action A_j. The payoffs delivered on each trial depended on both the action chosen by the decision maker and a binary event randomly sampled from a Bernoulli process (labeled here for convenience as Events X and Y).

Design

Table 2 shows the payoffs produced by each action-event pair for each of the five conditions. For each action, one of two possible payoffs was delivered on each trial, depending on the outcome of the binary event. For example, in the first condition, if A_1 was chosen and Event X occurred, then .10 DM units of money was earned and a 1-s burst of noise was delivered to the ears; if A_1 was chosen and Event Y occurred, then –.05 DM was received and a 10-s burst of noise was delivered to the ears.

The conditions differed according to the probabilities and payoffs. The two events were equally likely for Conditions 1 through 3; in Conditions 4 and 5, the probabilities were biased, by two different amounts, in the direction favoring the stochastically dominant alternative. Both an increase in money and a decrease in noise were used to produce the dominant action in Condition 1; only the tone was changed to form the dominant action in Conditions 2, 4, and 5; and only money was changed to form the dominant action in Condition 3.

Table 2. *Payoffs in each condition*

Action	Event X	Event Y
	Condition 1: *Pr*(X) = .50, *Pr*(Y) = .50	
A	.10 DM, 1 s	–.05 DM, 10 s
B	–.10 DM, 15 s	.05 DM, 5 s
C	.05 DM, 5 s	–.10 DM, 15 s
	Condition 2: *Pr*(X) = .50, *Pr*(Y) = .50	
A	.10 DM, 1 s	–.05 DM, 10 s
B	–.05 DM, 15 s	.10 DM, 5 s
C	.10 DM, 5 s	–.05 DM, 15 s
	Condition 3: *Pr*(X) = .50, *Pr*(Y) = .50	
A	.10 DM, 1 s	–.05 DM, 10 s
B	–.10 DM, 10 s	.05 DM, 1 s
C	.05 DM, 1 s	–.10 DM, 10 s
	Condition 4: *Pr*(X) = .60, *Pr*(Y) = .40	
A	.10 DM, 1 s	–.05 DM, 10 s
B	–.05 DM, 15 s	.10 DM, 5 s
C	.10 DM, 5 s	–.05 DM, 15 s
	Condition 5: *Pr*(X) = .70, *Pr*(Y) = .30	
A	.10 DM, 1 s	–.05 DM, 10 s
B	–.05 DM, 15 s	.10 DM, 5 s
C	.10 DM, 5 s	–.05 DM, 15 s

Note. Seconds refer to the duration of a noisy tone burst. 1 DM (Deutsche mark) equals approximately $0.80.

As can be seen from Table 2, A_j stochastically dominated both B_j and C_j for all conditions, the payoffs for B_j were negatively correlated with the payoffs for A_j, and the payoffs for C_j were positively correlated with the payoffs for A_j. Therefore, conflict was expected to be produced by the choice pair (A_j, B_j), but no conflict was expected to be produced by the pair (A_j, C_j).

RESULTS

Choice Probabilities

Table 3 shows the choice probabilities and mean response times for the 2 subjects in the first part of the study. Results for the five conditions and means averaged across conditions are shown. Each proportion within a condition was based on 240 observations and has a standard error less than .032, and each mean proportion at the bottom of the table was based on 1,200 observations and has a standard error less than .014.

Table 3 shows that the dominated action was never chosen under the positively correlated (no-conflict) condition,[1] a result consistent with the stochastic-dominance principle. But contrary to the stochastic-dominance principle, Table 3 shows that the dominated action was

1. The only exception was Condition 1 for subject J.B., which was the very first condition in the experiment for this subject. After this first condition, J.B.'s behavior consistently followed the general pattern.

Table 3. *Choice probabilities and mean response times for each condition and choice pair: Subjects H.G. and J.B.*

	Subject			
	H.G.		J.B.	
Choice pair and response[a]	Probability	Decision time (in seconds)	Probability	Decision time (in seconds)
Condition 1				
ABA	.85	0.31	.97	0.35
ABB	.15	0.58	.03	0.73
ACA	1.00	0.32	.93	0.34
ACC	0.00	—	.07	0.96
Condition 2				
ABA	.57	0.96	.57	0.25
ABB	.43	0.95	.42	0.72
ACA	1.00	0.71	1.00	0.45
Condition 3				
ABA	.71	0.45	.80	0.35
ABB	.29	0.73	.20	0.76
ACA	1.00	0.44	1.00	0.37
Condition 4				
ABA	.596	0.569	.821	0.305
ABB	.404	0.611	.179	0.838
ACA	1.00	0.469	1.00	0.345
Condition 5				
ABA	.567	0.353	.808	0.212
ABB	.433	0.552	.192	1.167
ACA	1.00	0.365	1.00	0.37
Mean				
ABA	.66	0.53	.79	0.29
ABB	.34	0.68	.21	0.61
ACA	1.00	0.46	.99	0.38

[a]In each three-letter notation, the first two letters indicate the choices presented, and the third letter indicates which action was chosen. Thus, ABA means choice pair (A, B) was presented and Action A was chosen, and ABB means choice pair (A, B) was presented and Action B was chosen.

frequently chosen in the negatively correlated (conflict) condition. The fact that these violations depended dramatically on the correlation between the payoffs is contrary to the predictions of the strong utility model of risky decision making.

Mean Response Times

Table 3 also shows that the mean response time was inversely related to the choice probability: The dominant action was chosen more quickly than the dominated action. However, there was little overall difference between the mean response times for the dominant actions under the positive-correlation (no-conflict) versus negative-correlation (conflict) conditions.

Results for Part 2

Table 4 shows the results pooled across the 17 subjects from the second part of the study. As can be seen in this table, the basic pattern of results from the first part of the study was replicated in the second part. Stochastic dominance was frequently violated under the high-conflict (negatively correlated) condition, but rarely violated under the no-conflict (positively correlated) condition. The difference between these two proportions was statistically significant, $t(16) = 4.01$, $p = .0005$.

Response Time Distributions

Figure 1 shows the cumulative distribution of choice times, pooled across the last two conditions.[2] Consider first the upper and lower curves in the top panels of Figure 1, produced by the dominant and dominated actions, respectively, in the negatively correlated (conflict)

2. Conditions 4 and 5 were used for this analysis because they were the last two conditions of the experiment for both subjects. These conditions were selected to minimize any possible training effects on response time. The distributions within each panel were based on 480 observations per subject and condition.

Table 4. *Choice probabilities and mean response times for 17 subjects in Condition 5*

Choice pair and response[a]	Probability	Decision time (in seconds)
ABA	.90	0.81
ABB	.10	1.52
ACA	.99	0.60
ACC	.01	1.79

[a]In each three-letter notation, the first two letters indicate the choices presented, and the third letter indicates which action was chosen. Thus, ABA means choice pair (A, B) was presented and Action A was chosen, and ABB means choice pair (A, B) was presented and Action B was chosen.

condition. The rise is much faster in the distribution produced by the dominant action than in the distribution produced by the dominated action. There are also substantial differences between the 2 participants across these two sets of curves.

Next, compare the curves for the dominant action in the negatively correlated (top panel) versus positively correlated (bottom panel) conditions. Surprisingly, there is very little difference between these two distributions for each subject. The only noticeable difference is an atypical acceleration that appeared at 0.40 s in the distribution for one subject (J.B.) under the positive-correlation condition.

MDFT (Diederich, 1997) was used to generate predictions for the response time distributions (see the appendix for details). A comparison of the predicted with the observed values indicates that the model provided only a marginal fit of the data. First, as can be seen within the legends (boxes) set inside Figure 1, the model accurately reproduced the choice probabilities for each subject and condition. Second, the model reproduced a more rapid rise in the distribution for the dominant compared with the nondominant action, and it also reflected the large individual differences observed for the negatively correlated condition. However, the model failed to capture the shift in the leading edge of the distribution of the nondominant action in the negatively correlated condition for subject H.G., and it failed to accurately capture the acceleration in the distribution for the nondominant action for subject J.B. In short, the model had more success fitting the distributions for the dominant actions and less success with some of the detailed aspects of the distributions for the nondominant actions.

DISCUSSION

One of the key principles underlying rational models of decision making is the idea that the decision maker should always choose a stochastically dominant action over a dominated action. Human nature does not always obey these rational principles, and empirical studies of human decision making have uncovered a number of surprising violations. For example, Birnbaum and Mellers (Birnbaum, Coffey, Mellers, & Weiss, 1992; Mellers, Weiss, & Birnbaum, 1992) have found special conditions in which decision makers are willing to pay a higher price for a stochastically dominated gamble compared with the price they would pay for a stochastically dominant gamble.

This study examined conditions under which decision makers directly chose between two uncertain actions, with one action stochastically dominating the other action. Violations of stochastic dominance frequently occurred when the payoffs produced by two actions were negatively correlated (high conflict), but no violations occurred when the payoffs were positively correlated (no conflict). This finding is contrary to strong utility models, which assume that the utility for an action depends solely on its own payoffs and probabilities (i.e., solely on one row of a payoff matrix).[3]

DFT (see Busemeyer & Townsend, 1993), and its recent generalization called MDFT (Diederich, 1997), predicted the violations of stochastic dominance reported in this article. According to this theory, the decision maker's attention changes from moment to moment, switching back and forth from one uncertain state to another during deliberation. While attending momentarily to a particular state, the individual compares consequences produced by each action under that state. These momentary comparisons are integrated over time to form an integrated preference for each action. When the payoffs are negatively correlated, the comparisons change sign back and forth from positive to negative as attention fluctuates, producing up-and-down vacillations in preference that lead to violations of stochastic dominance. When the payoffs are positively correlated, the comparison always produces a positive (or zero) increment favoring Action A over Action B, independent of the state to which the decision maker attends. In this case, preference for the dominant action always increases over time, so that stochastic dominance is satisfied.

Other psychological frameworks that allow conflict to influence the decision process (e.g., Janis & Mann, 1977; Mellers, Schwartz, & Ritov, 1997; Svenson, 1992; Tversky & Shafir, 1992) may be able to explain these findings in a post hoc manner. However, the advantage of DFT over these other frameworks is that only DFT is able to provide a priori predictions for the effects of payoff correlation on frequency of stochastic-dominance violations.

The present experiment also provided a first look at the distribution of response times for binary choices between risky courses of action. The absence of conflict permits the use of a decision process that is simpler than the decision process used to make decisions under conflict. This observation suggests that there might be dramatically different distributions for the positive- versus negative-correlation conditions. Surprisingly, the distribution of choice times for the dominant action was approximately the same for the negatively correlated (conflict) and positively correlated (no-conflict) conditions.

MDFT uses a common decision process for both the negatively correlated (high-conflict) and the positively correlated (no-conflict) conditions. A quantitative test of this model was performed by evaluating its predictions for the distributions of choice response time in the presence or absence of conflict. The fits of the model were marginal, and the model encountered some difficulty reproducing the details of the response time distributions for nondominant actions. However, recognizing that this is the first attempt to fit choice response time distributions obtained from a risky decision task, we conclude that the theory is worthy of further consideration.

In summary, although stochastic dominance remains a viable principle of rational decision making, it is frequently violated empirically for psychological reasons. Therefore, psychological theories of deci-

3. These results were obtained using a procedure under which subjects actually experienced the payoffs on each trial (money and tone burst) and learned the probabilities of each outcome from experience. Future research is needed to see whether the same results will be obtained if fictitious payoffs are used and probabilities are described verbally or textually.

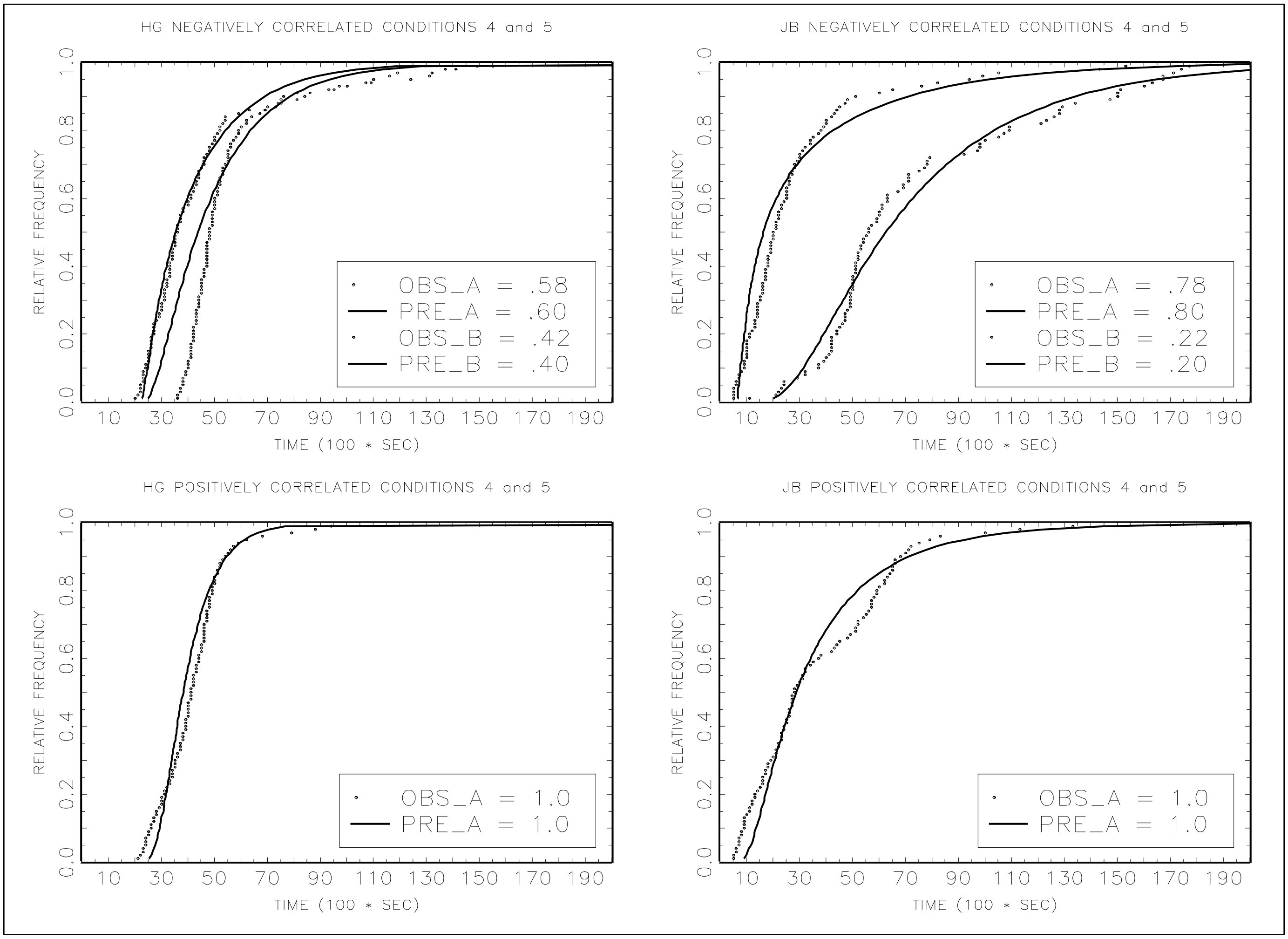

Fig. 1. Distribution of response times for subject H.G. (left panels) and subject J.B. (right panels) under the negatively correlated (top panels) and positively correlated (bottom panels) payoff conditions. The observed ("OBS") data are represented by the plotted points, and the predictions ("PRE") of the multiattribute decision field model are represented by the smooth curves. Within the top panels, the upper curve shows the distribution of times required to choose the dominant action (A), and the lower curve shows the distribution of times required to choose the dominated action (B). Within the bottom panels, each curve shows the distribution of times to choose the dominant action (A). The box in each panel shows the observed and predicted choice probabilities.

sion making need not be constrained to satisfy this principle; instead, they need to predict when violations of stochastic dominance will occur.

Acknowledgments—This research was supported by grants from the National Institute of Mental Health (Perception and Cognition R01 MH55680) and the National Science Foundation (Decision Risk Management Science SBR-9602102) to the first author.

REFERENCES

Birnbaum, M.H., Coffey, G., Mellers, B.A., & Weiss, R. (1992). Utility measurement: Configural-weight theory and the judge's point of view. *Journal of Experimental Psychology: Human Perception and Performance, 18,* 331–346.

Birnbaum, M.H., & Thompson, L.A. (1996). Violations of monotonicity in choices between gambles and certain cash. *American Journal of Psychology, 109,* 501–523.

Busemeyer, J.R., & Townsend, J.T. (1993). Decision field theory: A dynamic-cognitive approach to decision making in an uncertain environment. *Psychological Review, 100,* 432–459.

Clemen, R.T. (1996). *Making hard decisions.* New York: Duxbury Press.

Diederich, A. (1997). Dynamic stochastic models for decision making under time pressure. *Journal of Mathematical Psychology, 41,* 260–274.

Diederich, A., & Busemeyer, J.R. (1999). *Multi-attribute decision field theory: A dynamic-cognitive approach to multi-attribute risky decision making.* Unpublished manuscript, Univesitat Oldenburg, Oldenburg, Germany.

Janis, I., & Mann, L. (1977). *Decision making: A psychological analysis of conflict, choice, and commitment.* New York: Free Press.

Luce, R.D. (1992). Where does subjective expected utility theory fail descriptively? *Journal of Risk and Uncertainty, 5,* 5–28.

Luce, R.D., & Suppes, P. (1965). Preference, utility, and subjective probability. In R.D. Luce, R.R. Bush, & E. Galanter (Eds.), *Handbook of mathematical psychology* (Vol. 3, pp. 249–410). New York: Wiley.

Mellers, B.A., Schwartz, A., Ho, K., & Ritov, I. (1997). Decision affect theory: How we feel about risky options. *Psychological Science, 8,* 423–429.

Mellers, B.A., Weiss, R., & Birnbaum, M.H. (1992). Violations of dominance in pricing judgments. *Journal of Risk and Uncertainty, 5,* 73–90.

Ratcliff, R., & Rouder, J.N. (1998). Modeling response times for two-choice decisions. *Psychological Science, 9,* 347–356.

Svenson, O. (1992). Differentiation and consolidation theory of human decision making: A frame of reference for the study of pre- and post-decision processes. *Acta Psychologica, 80,* 143–168.

Townsend, J.T., & Busemeyer, J.R. (1995). Dynamic representation of decision making. In R.F. Port & T. van Gelder (Eds.), *Mind as motion* (pp. 101–121). Cambridge, MA: MIT Press.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty, 4,* 297–324.

Tversky, A., & Shafir, E. (1992). Choice under conflict: The dynamics of deferred decision. *Psychological Science, 3,* 358–361.

Wakker, P. (1989). Transforming probabilities without violating stochastic dominance. In E.E. Roskam (Ed.), *Mathematical psychology in progress* (pp. 29–47). Berlin: Springer-Verlag.

(RECEIVED 5/15/98; REVISION ACCEPTED 1/26/99)

APPENDIX

The predictions in Figure 1 were derived from the first-passage-time distribution for preference state $P(t)$ to cross the threshold bound θ using Markov chain theory. Because of the brevity of this report, it is not possible to describe the Markov chain solution; however, these equations are available in Diederich (1997). Not all of the parameters of MDFT were required to fit the data for this particular design. For example, only the ratio of the mean and standard deviation of the valence was required, so the variance was fixed equal to 1.0, and the decay rate was fixed to $s = 0$ for simplicity.

The following parameters were estimated by minimizing the sum of squared deviations between the predicted and observed percentiles in Figure 1: The mean valences for the two attributes were (1.92, 0.0) for H.G. and (1.35, 0.0) for J.B., the threshold bound was .7 for H.G. and 1.0 for J.B., and the starting preference was .08 for H.G. and .59 for J.B. The model assumed that subjects initially attend to the most important dimension for a stochastically determined period of time, and if a decision is not reached during this first period of time, attention switches to the less important dimension until a decision is reached. The probability of switching from the more important to the less important attribute at each time step was .19 for H.G. and .12 for J.B. According to the model, the main difference between the 2 subjects was the relatively large initial starting preference for subject J.B. compared with H.G. This initial starting preference represents the effect of memory for preferences from previous trials recalled at the beginning of the current trial.