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COMMENT

Theoretical Developments in Decision Field Theory: Comment on Tsetsos, Usher, and Chater (2010)

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Tsetsos, Usher, and Chater (2010) presented several criticisms of decision field theory (DFT) involving its distance function, instability under externally controlled stopping times, and lack of robustness to various multialternative choice scenarios. Here, we counter those claims with a specification of a distance function based on the indifference and dominance dimensions. Using this distance function, we show that the instability problems do not arise when using the internally controlled stopping rule. In conclusion, we argue that the predictions of DFT do not conflict with the data presented and that the model yet provides a coherent and accurate account of multialternative choice phenomena.

Keywords: choice behavior, decision making, models, preferences, distance function

Cognitive models of decision making are still relatively new developments in the study of human choice. These models seek to explain behavior as it arises from the collaboration of basic psychological processes, rather than through the satisfaction of rational axioms. To date, decision field theory (DFT) has been among the most successful cognitive models of decision making and has been applied to a broad range of phenomena including decision making under uncertainty, decision time, preference reversals between choice and prices, and context effects (Busemeyer & Townsend, 1993; Johnson & Busemeyer, 2005; Roe, Busemeyer, & Townsend, 2001). Since DFT's inception, several competing models have been proposed, including the leaky competing accumulator model (LCA; Usher & McClelland, 2004). Despite its many similarities to DFT, LCA differs in some key aspects. Tsetsos, Usher, and Chater (2010) presented several criticisms of DFT and its underlying mechanisms. In this article, we respond to those criticisms and introduce important theoretical developments by way of a more sophisticated distance function. We believe that these new developments overcome many of the problems put forward by Tsetsos et al. We also give our own critique of the studies they presented.

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Distance-Dependent Inhibition in DFT

Roe et al. (2001) did not specify a parametric form for the distance function for determining lateral inhibition between competing alternatives used in the DFT model. As pointed out by Tsetsos et al. (2010), this is needed to make predictions for a wide range of points in the economy–quality space of options. We propose a measure based on the intuition that people will quickly discard or ignore clearly dominated options. The basic idea is that inhibition decreases more slowly along the line of indifference (the negative slope in the economy–quality space) and that inhibition decreases more rapidly along the line of dominance (the positive slope in the economy-quality space, orthogonal to the line of indifference, where alternatives rapidly become irrelevant because they are defective).

The line of indifference is in the direction of the vector $(-1, 1)/\sqrt{2}$, and the line of dominance is in the direction of the vector $(1, 1)/\sqrt{2}$ (see the direction vectors in Figure 1). Suppose an option is described by the two coordinates (E = economy, Q = quality). Consider the three options A = (1, 3), B = (2, 2), and C = (0, 2). The change from A to B is a change along the indifference line and these two options are highly competitive, but the change from A to C is along the dominance line and C is defective. The difference between A and B is $(1, 3) - (2, 2) = (-1, 1)$, and the difference between A and C equals $(1, 3) - (0, 2) = (1, 1)$. We do not think these two changes should be treated the same and assigned the same distance. We argue that distance increases more slowly in the former (competitive) case when the change is along the line of indifference and that it changes more rapidly in the latter (dominated) case when the change is along the line of dominance.

This idea is mathematically expressed by using the following distance function. Define $(\Delta E, \Delta Q)$ as a difference between two options described by the economic and quality directions. Then, define $\Delta I = (\Delta Q - \Delta E)/\sqrt{2}$ and $\Delta D = (\Delta Q + \Delta E)/\sqrt{2}$, which are the corresponding coordinates of this option with respect to the indifference and dominance directions. For example, the differ-

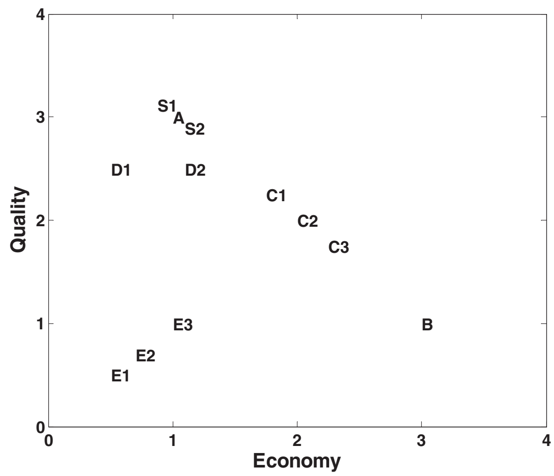

Figure 1. Various choice alternatives in a two-dimensional space. The indifference and dominance dimensions are also displayed.

ence between Options A and B can be expressed as $(2, 0)/\sqrt{2}$ with respect to the indifference and dominance directions. The difference between Options A and C can be expressed as $(0, 2)/\sqrt{2}$ with respect to the indifference and dominance directions. If (E1, Q1) and (E2, Q2) are two options described by economy and quality, then $(\Delta E, \Delta Q) = (E1 - E2, Q1 - Q2)$ and $(\Delta I, \Delta D) = [(\Delta Q - \Delta E), (\Delta Q + \Delta E)]$. The distance function should weight changes in the dominance direction more than in the indifference direction. This is achieved by applying a larger weight $b > 1$ to the dominance direction:

$$D_{ij} = \text{Dist}(\text{Option\_}i, \text{Option\_}j) = \text{Dist}[(E_i, Q_i), (E_j, Q_j)]$$

$$= \text{Dist}[(\Delta I, \Delta D)] = (\Delta I)^2 + b \times (\Delta D)^2.$$

For example, suppose $b = 10$, then Dist(A, B) = 2, Dist(A, C) = 20, and Dist(B, C) = 22.

The distance can then be mapped into the feedback **S** matrix (self feedback when $i = j$ and $\delta_{ij} = 1$, lateral inhibition when $i \neq j$ and $\delta_{ij} = 0$) by the Gaussian function:

$$s_{ij} = \delta_{ij} - \varphi_2 \times \exp(-\varphi_1 \times D_{ij}^2).$$

For example, setting $\varphi_1 = .01$, $\varphi_2 = .10$, and $b = 10$ produces $s_{AB} = -.0961$, $s_{AC} = -.0018$, and $s_{BC} = -.0008$. When $i = j$, we obtain $s_{ii} = 1 - \varphi_2 = 1 - .10 = .90$. The DFT model then has six parameters: the attention weight, w_Q, given to quality; the attention weight, $w_E = 1 - w_Q$, given to economics; the variance, σ^2, of the noise; and the three parameters, φ_1, φ_2, b, for the S matrix. When the internally controlled paradigm is used, we have a threshold bound. When the externally controlled paradigm is used, we have a parameter for shutting off attention to task.

We evaluated the predictions of DFT produced by this new distance function using two different stopping rules. First, we examined the internally controlled stopping rule, which is appropriate for situations in which the decision maker determines when to stop and make a decision. In this case, we assume that preferences are accumulated until a threshold criterion is reached and that the first option to reach the threshold is chosen. Later, we reexamined the predictions using the externally controlled stopping rule. This rule is appropriate for situations in which the decision maker deliberates for a fixed time, at which point the alternative with the maximum preference is chosen. We focus more on the internally controlled stopping rule because this is more common; however, Tsetsos et al. (2010) focused more on the externally controlled stopping rule.

Assume that the binary choice probability for any two items in the set is .50. Thus each of these alternatives is on the indifference line (see Figure 1). These binary choice probabilities indicate that the attention weight for quality equals the weight for economics so that $w_Q = w_E = .50$. We arbitrarily set $\sigma = 1$ and adjusted the threshold for this level of noise, and we found $\theta = 17.5$ produced appropriate results. The remaining parameters we set equal to $\varphi_1 = .022$, $\varphi_2 = .05$, and $b = 12$. We then computed the predictions for the 18 triads shown in Table 1 (also see Figure 1). These predictions were computed by simulating the internally controlled stopping rule process for 20,000 simulations (see the program on Jerome R. Busemeyer's website[1]). As can be seen in Table 1, the model produces all the expected results. There does not seem to be any strange problem for the three compromise conditions. Also, the results for the E1, E3, C2 triad and the E1, E2, C2 triad clearly show that there is no problem for DFT to produce the right predictions for these dominated options.

Finally, consider the four options shown in Table 2 (ignore values in parentheses at this point). We calculated the predicted choice probabilities for the internally controlled stopping rule and using the same parameters as described earlier. For $\theta = 20$, we obtained $P(A) = .44$, $P(B) = .56$, and $P(C) = P(D) = 0$. Mean response time was 129 time steps. Results were similar for other values of θ, with $P(B)$ increasing at the expense of $P(A)$ as θ increased. These results show a preference for Option B over A and no choices for the dominated options.

Robustness of the Compromise Effect in DFT

Tsetsos et al. (2010) raised concerns that the distance between extreme alternatives determines whether or not DFT predicts the compromise effect. This analysis overlooks the fact that the attribute space has an arbitrary scale. The failure of DFT to produce the effect with very small or very large distances could be remedied by simply rescaling the space.

Tsetsos et al. (2010) also offered simulations showing that the compromise effect can reverse in DFT for five-alternative choice problems. We believe that an individual faced with this choice set would not distinguish between similar extreme alternatives but would instead deliberate between three groups of options: (a) a high-quality, low-economy group; (b) a low-quality, high-economy group; and (c) compromise. This grouping of similar alternatives makes the compromise the preferred option.

Testing Correlations Between Preferences in DFT

Tsetsos et al. (2010) presented data from an experiment conducted by Usher, Elhalal, and McClelland (2008) in which participants chose between three alternatives, with one being a compromise between the other two. The results showed that when an

[1] http://mypage.iu.edu/~jbusemey/MDFT_wgt_selection.txt

Table 1
Results Predicted by Decison Field Theory With Internally Controlled Stopping Rule

Triad	First	Second	Third	Mean time
A, B, S1 similarity	.30	.40	.30	143
A, B, S2 similarity	.31	.38	.31	148
A, B, C1 compromise	.28	.30	.42	116
A, B, C2 compromise	.28	.28	.43	111
A, B, C3	.29	.28	.42	115
A, B, D1 attraction	.56	.44	0	84
A, B, D2 attraction	.62	.37	.01	83
E1, E3, C2 dominance	0	0	1	20
E1, E2, C2 dominance	0	0	1	24

extreme option was chosen and then announced to be unavailable, participants tended to choose the compromise from the remaining options. This appeared contrary to DFT's prediction—based on the correlation of preference states for extreme options—that they should choose the other extreme. However, this study involves a potential selection bias whereby individuals who initially chose an extreme were likely attending to one attribute dimension more than another. In such cases of asymmetric attention weighting, DFT predicts that the compromise will be preferred for the second choice, whether or not deliberation is restarted after the first choice, because it is superior on the more heavily weighted dimension. We tested this hypothesis by assuming that individuals' attention weights are described by $w_Q = \rho \times .5 + (1 - \rho) \times \varepsilon$ and $w_E = 1 - w_Q$, where ε is generated from a uniform random distribution between 0 and 1, and $0 < \rho < 1$ determines the extent of the variation across individuals. All other model parameters were set equal to those used in the above simulations. For intermediate values of ρ (i.e., 0.47–0.75), DFT does indeed predict the compromise effect for the trinary choice and that the compromise will be preferred for the second choice if an extreme alternative is removed from competition after reaching the decision threshold. Furthermore, it is difficult to know how exactly to model the data reported in Usher et al. because they did not report binary choice probabilities for the three alternatives. It is therefore possible that the alternatives did not lie on the indifference line of psychological attribute space. The large compromise effect in the first choice suggests that the compromise option may have been superior to the extremes in binary choice. Also, it is possible that one of the extreme alternatives, A, was significantly inferior to the other, B, in binary choice. If so, this inferior alternative would stand little chance against the compromise after B was chosen and announced to be unavailable.

As pointed out in Tsetsos et al. (2010), Busemeyer and Johnson (2004) proposed another mechanism that would account for this result within the DFT framework. If an alternative becomes unavailable and the person is prevented from choosing it, then this availability can be thought of as a third attribute, with unavailable alternatives having a very low value on this dimension. If the unavailable option continues to compete for selection, its low value on availability will give it negative activation. This would, in turn, boost its nearest competitor, which is the compromise. Tsetsos et al. countered this explanation with an experiment where decision makers were given three alternatives arranged to produce an attraction effect. In some conditions the dominated decoy was announced to be unavailable after 15 s of deliberation, after which individuals waited another 15 s before choosing between the remaining items. Tsetsos et al. found that when the dominated decoy was made unavailable, choice probabilities return to the baseline for the binary choice, suggesting that the unavailable decoy did not compete. However, there is surely a limit to the amount of time that an unavailable option remains in contention. Fifteen s may have been too long of a delay to elicit the phantom attraction effect. This would not be true of the Usher et al. (2008) study, where choices were speeded.

Table 2
A Four-Alternative Choice Problem

Choice	Economics	Quality
A	1	3
B	3	1
C	1	2 (2.9)
D	1	1 (2.8)

Source of the Instability Problem

Tsetsos et al. (2010) pointed out the possibility that DFT's dynamics can become unstable and send preference states to±infinity under long external stopping times. This instability problem arises from trying to fit all nine conditions in Table 1 using a common parameter for the variance of the noise. Noise variance comes from attention to dimensions not under experimenter control (irrelevant dimensions with respect to the design of the experiment). This is not reasonable because the experiments that Tversky (1972) used to study similarity were based on very simple and highly controlled stimuli (producing low noise variance) but the stimuli used to study attraction and compromise effects are generally much more complex (which produces higher noise variance). Suppose we set $\sigma = .05$ to fit the first two (similarity) rows of Table 1 and we set $\sigma = 1$ for the remaining (attraction, compromise, dominance) rows. Now, we can set $s_{ij} = (.99) \times (\delta_{ij} - \varphi_2 \times \exp[-\varphi_1 \times D_{ij}^2])$ with $\varphi_1 = .022$, $\varphi_2 = .05$, and $b = 12$, as before. The eigenvalues of the **S** matrix are all sufficiently far below 1 that the system is highly stable. The predictions for time $T = 1{,}001$ using the externally controlled stopping rule are given in Table 3 (the same results obtain with $T = 2{,}001$). These results essentially match those shown in Table 1.

Table 3
Decison Field Theory With Experimenter-Controlled Stopping Rule at Time $T = 1{,}001$

Triad	First	Second	Third	σ
A, B, S1 similarity	.26	.44	.30	.05
A, B, S2 similarity	.30	.44	.26	.05
A, B, C1 compromise	.31	.26	.42	1
A, B, C2 compromise	.28	.28	.45	1
A, B, C3	.26	.31	.42	1
A, B, D1 attraction	.67	.33	0	1
A, B, D2 attraction	.77	.22	.01	1
E1, E3, C2 dominance	0	.03	.97	1
E1, E2, C2 dominance	0	0	1	1

Stopping Rules and Stability

In addition to the strategy laid out above, there are two ways to stabilize the system within DFT. For an internally controlled stopping task, we assume that the decision maker chooses a threshold bound and stops the accumulation process as soon as an option reaches the threshold. Such cases, which are surely the majority of both real-world and laboratory choice scenarios, do not produce instability problems in DFT (see Table 1).

For the externally controlled stopping process, the decision maker is asked to make a choice at a fixed time, *T,* and in this case, we assume information is accumulated until the fixed stopping time, at which point the option with the maximum preference is chosen. We argue that Tsetsos et al. (2010) examined relatively long fixed stopping times in their analyses (relative to the stopping times needed for the results shown in Table 1). Although it is true that the physical duration of a simulated time step is not known without fitting response times (M. Usher, personal communication, November 1, 2009), the number of steps they used far exceeds those predicted by the model for internally controlled stopping (e.g., the above simulation in Table 1 found the mean number of steps was 129 time steps, which is far smaller than the number of time steps used by Tsetsos et al., 2010, to demonstrate instability). Furthermore, if the information is fixed and the time to make the decision is very long, the decision maker may stop deliberating and simply wait for the appointed decision time. In other words, the person may make up his or her mind much earlier than the deadline. If we set the fixed time limit to $T = 202$ (as done in Roe et al., 2001), which is even longer than the mean time to make a decision shown in Table 1 with the internally controlled stopping rule, then the predictions from the analytic solutions for the externally controlled stopping rule (the equations given in Appendix B of Roe et al., 2001) reproduce the results shown in Table 1, demonstrating the robustness of DFT's predictions.

The psychological intuition is that the decision maker is not going to deliberate forever even when asked to do so. If there is no new information, then the decision maker will quit deliberating after a while and just retain the current preference state. Of course, if new information is presented, the decision maker may start paying attention again and resume deliberation until the new information has been digested.

There are, however, some extreme cases where problems might still arise for DFT. For example, consider the extreme case shown in Figure 6a of Tsetsos et al. (2010) where $C > B > A$, but A and B are so close together (relative to the distance from C) that they become indistinguishable. In these extreme cases, we assert that decision makers simply group the highly similar options together, and so, this would actually be treated as a binary choice between the dominant option (C) and the dominated group (A and B together). In this case, the binary choice model would always choose the dominant option. This grouping of almost indistinguishable options could be modeled more systematically by allowing the maximum lateral inhibition to arise at small nonzero distances and then assuming that it decays toward zero as distance increases in magnitude as well as when distance approaches zero. The decay may not be symmetric around the maximum, and it may occur more rapidly as the distance approaches zero.

Stability problems are the price paid for DFT's relatively simple linear dynamics. Linear dynamics have the advantage of allowing one to derive analytical solutions for choice probabilities with the externally controlled stopping time (see Appendix B in Roe et al., 2001), and to derive analytic solutions for choice probabilities and choice response times for binary choices with the internally controlled stopping time (see the Appendix of Busemeyer & Townsend, 1993). Adopting nonlinear dynamics could solve these remaining stability problems but at the cost of losing the ability to derive analytic solutions, as with LCA.

Conclusion

The new distance model presented here provides a satisfactory explanation for all the main choice patterns observed in multialternative choice. Thus, the challenge posed by Tsetsos et al. (2010) about the existence of a simple distance model for determining inhibition in the DFT model has been answered in the affirmative (in fact, Tsetsos et al., 2010, themselves discovered an alternative distance function for DFT that addresses this challenge). We argue that it is critical to use a measure that weights changes along the dominance direction more than changes along the indifference direction. This makes sense because people will quickly discard or ignore clearly dominated options. Also, the internally controlled stopping rule used in DFT avoids instability problems that can arise by using a linear dynamical preference evolution that accumulates to a threshold. We have also raised questions about the experiments used to challenge DFT's explanation for context effects and have attempted to show why these data do not conflict with the predictions of the model. In light of these points, we maintain that DFT gives a robust account of multialternative choice. Also, it is important to note that DFT was developed to model many other phenomena in addition to multialternative choice findings. These include decision times, time pressure, buying and selling prices, and dynamic inconsistencies. Although LCA is applicable to many of these domains, its predictions have been less widely tested.

Stepping back and taking a broader view, we argue that the commonalities between DFT and LCA are much greater than their differences. Both are built on firm cognitive principles, and consequently, they share many advantages for understanding human decision making when compared to mainstream economic utility models. In sum, we thank the authors for their fair and thoughtful criticism that has stimulated advances in both models. It is through this kind of debate that scientific progress is made.

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Correction to Baumeister and Masicampo (2010)

In the article "Conscious Thought Is for Facilitating Social and Cultural Interactions: How Mental Simulations Serve the Animal-Culture Interface" by Roy F. Baumeister and E. J. Masicampo (*Psychological Review*, 2010, Vol. 117, No. 3, 945-971), there was an error in the quotation on page 950. The sentence should have read: William James's dictum that *thinking is for doing* (James, 1890, Vol. 2, p. 333) has reigned as a truism in psychology for more than a century.

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